## SOLAR ENERGY

How much strikes the earth?
How much can my house get?

ENGS-44 Sustainable Design
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The amount of incident solar energy on earth each year
= 160 times the world's proven resources of fossil fuels
$=1,500$ times the world's annual energy use.

Three mechanisms of heat transfer:

- conduction (molecular agitation in the material)
- convection (movement of carrying fluid)
- radiation (electromagnetic waves)


Solar energy is carried across empty space from the sun to the earth by radiation of electromagnetic waves (infra-red, visible \& ultra-violet).

Most of this radiation is in the visible spectrum, to which the atmosphere is quite transparent.

## Two basic laws of heat radiation:

1. All objects emit radiation. The hotter they are, the more they radiate.

The emitted radiation flux (energy per unit area and unit time), $E$, is given by:

$$
E=\sigma T^{4}
$$

where $\quad T=$ absolute temperature in degree Kelvin (= ${ }^{\circ} \mathrm{C}+273.15$ )

$$
\sigma=5.67 \times 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2} . \mathrm{K}^{4}\right)
$$

or where $\quad T=$ absolute temperature in degree Rankine ( $={ }^{\circ} \mathrm{F}+524$ ) $\sigma=1.71 \times 10^{-9} \mathrm{Btu} /\left(\mathrm{ft}^{2} . \mathrm{hr} . \mathrm{R}^{4}\right)$
2. The radiation emitted by a body at absolute temperature $T$ fills a spectrum, with peak at wavelength $\lambda$ given by:

$$
\lambda=\frac{2898 \mu \mathrm{~m} \cdot K}{T(\mathrm{in} \mathrm{~K})}=\frac{5216 \mu \mathrm{~m} \cdot R}{T(\mathrm{in} \mathrm{R})}
$$

Thus, the hotter the body, the shorter the emitted wavelengths.

## Consequence of Rule 1:

One way to provide heat to an object is to expose it to a hotter body. The hotter, the better. Hence, solar exposure is much better than exposure to a warm piece of earth or even a fire.

## Consequence of Rule 2:

Because the sun is so hot ( $T=5750 \mathrm{~K}=9,890^{\circ} \mathrm{F}$ ), it emits most of its radiation around $\lambda=0.50 \mu \mathrm{~m}$, which not coincidentally falls in the visible range.

The earth and our houses are not as hot (around $T=72^{\circ} \mathrm{F}=295 \mathrm{~K}$ ) and emit their radiation around $\lambda=10 \mu \mathrm{~m}$, in the infra-red range. We need an infra-red camera to "see" this radiation.

Radiation from sun and from earth


The atmosphere and window glass are mostly transparent to visible light but quite opaque in the infra-red range.

From sun to earth:

Being at 5750 K , the sun emits $6.2 \times 10^{7} \mathrm{~W} / \mathrm{m}^{2}$.
Given the size of the sun ( $R_{\text {sun }}=696,000 \mathrm{~km}$ ) and the distance from the sun to the earth ( $d=149,476,000 \mathrm{~km}$ ), we can calculate the amount of the solar radiation arriving at the earth:

$$
1372 \mathrm{~W} / \mathrm{m}^{2}=435 \mathrm{Btu} /\left(\mathrm{hr} . \mathrm{ft}^{2}\right)
$$

at normal incidence.
The preceding figure is the solar radiation arriving at the outer edge of the earth, which is the upper atmosphere. What actually strikes the earth surface is somewhat less because of partial absorption and reflection by the atmosphere, especially clouds (in average, about 60\% left at ground level).

A further reduction is caused by oblique incidence (radiation spread over a larger area).


Value of the incident solar radiation flux $I$ at ground level?

The value of $I$ varies with

- Latitude (solar declination, more tangential at high latitudes)
- Length of path through the atmosphere (oblique incidence)
- Climate (cloudiness factor)

|  | Month | $A\left(\right.$ (Btu/ft ${ }^{2}$.hr) | $B$ |
| :---: | :---: | :---: | :---: |
|  | Jan | 390 | 0.142 |
| On a clear day: | Feb | 385 | 0.144 |
|  | Mar | 376 | 0.156 |
| $I_{\text {clear sky }}=A e^{-B / \sin \alpha}$ | Apr | 360 | 0.180 |
|  | May | 350 | 0.196 |
| where $\alpha=$ solar altitude | Jun | 345 | 0.205 |
| (angle of sun above horizon). | Jul | 344 | 0.207 |
|  | Aug | 351 | 0.201 |
|  | Sep | 365 | 0.177 |
|  | Oct | 378 | 0.160 |
|  | Nov | 387 | 0.149 |
|  | Dec | 391 | 0.142 |

The angle $\alpha$ of the sun above the horizon, at any given place and time, depends on 3 variables:


The trigonometric formula is:
$\sin \alpha=\sin \varphi \sin \delta+\cos \varphi \cos \delta \cos \left(15^{\circ} h\right)$
in which $\delta$ is the solar declination (angle of sun above equatorial plane):

$$
\delta=23.5^{\circ} \cos \left(360^{\circ} \frac{n-172}{365}\right)
$$



Look for key values of $\alpha$ :
$\sin \alpha=\sin \varphi \sin \delta+\cos \varphi \cos \delta \cos \left(15^{\circ} h\right)$

Over the course of the day, the sun is highest at noon $(h=0)$ :

$$
\begin{aligned}
\sin \alpha & =\sin \varphi \sin \delta+\cos \varphi \cos \delta \\
& =\cos (\varphi-\delta) \\
& =\sin \left(90^{\circ}-\varphi+\delta\right) \rightarrow \alpha=90^{\circ}+\delta-\varphi
\end{aligned}
$$

in which

$$
\delta=23.5^{\circ} \cos \left(360^{\circ} \frac{n-172}{365}\right)
$$

| Spring equinox (22 March): | $n=81 \rightarrow \delta=0$ | $\rightarrow \alpha=90^{\circ}-\varphi$ |
| :--- | :--- | :--- |
| Summer solstice (21 June): | $n=172 \rightarrow \delta=+23.5^{\circ}$ | $\rightarrow \alpha=113.5^{\circ}-\varphi$ |
| Fall equinox (20 September): | $n=263 \rightarrow \delta=0$ | $\rightarrow \alpha=90^{\circ}-\varphi$ |
| Winter solstice (21 December): | $n=355 \rightarrow \delta=-23.5^{\circ}$ | $\rightarrow \alpha=66.5^{\circ}-\varphi$ |


for $\varphi \geq 23.5^{\circ}$
So, if you know the height $H$ of the ceiling, you can calculate the length $L$ of overhang and depth $D$ of the room.

(http://www.strawbalehomes.com/solar1.html)

This house under construction in Durango, Colorado features straw-bale construction (excellent insulation) and passive solar design. Note the shade provided by the overhangs. Clerestories (in-roof windows) provide solar radiation to the back of the house.


The sun does not just move up and down in the sky, it also moves across the sky, rising in the East and setting in the West ... and occasionally not setting at all.


Island of Loppa, $70^{\circ} \mathrm{N}$, North Norway, 21-22 July (Credit: Husmo foto, Boks 231)

Taking into account the azimuth angle (sweep angle)
into account is complicated.


Skydome and associated cylindrical sun chart (Mazria, 1979)

Sun paths drawn on cylindrical sun chart. These are published.


Sun chart for $40^{\circ} \mathrm{N}$ (Mazria, 1979)

Adding the obstructions (॰), including the seasonal ones ( )


Finally, blocking the undesirable solar incidences of the summer


Solar radiation chart overlayed on solar window (Mazria, 1979)

For a first estimate (not adjusting for vegetation), one simply distinguishes between east, south, west and north facing walls of the structure, and use a so-called Solar Heat Gain Factor (SHGF) for each side.

For $40^{\circ} \mathrm{N}$, SHGF values
(in BTUs per $\mathrm{ft}^{2}$ per day, for average cloudiness in the USA):

In practice, use local cloudiness factor

| Month | \# days | East | South | West | North | \% sun |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| January | 31 |  | 452 | 1,626 | 452 | 0 |
| February | 28 | 648 | 1,642 | 648 | 0 | $46 \%$ |
| March | 31 | 832 | 1,388 | 832 | 0 | $55 \%$ |
| April | 30 | 957 | 976 | 957 | 0 | $56 \%$ |
| May | 31 | 1,024 | 716 | 1,024 | 0 | $57 \%$ |
| June | 30 | 1,038 | 630 | 1,038 | 0 | $60 \%$ |
| July | 31 | 1,008 | 704 | 1,008 | 0 | $62 \%$ |
| August | 31 | 928 | 948 | 928 | 0 | $60 \%$ |
| September | 30 | 787 | 1,344 | 787 | 0 | $57 \%$ |
| October | 31 | 623 | 1,582 | 623 | 0 | $55 \%$ |
| November | 30 | 445 | 1,596 | 445 | 0 | $46 \%$ |
| December | 31 | 374 | 1,114 | 374 | 0 | $46 \%$ |

Then apply a Shade Coefficient (multiply by 0.88 ) to account for partial reflection by glass if sunlight is captured inside of a window.

A small correction, often skipped:
Multiply previous values by this geographic factor to account for atmospheric clarity:


Estimated atmospheric clearness factor in the United States for nonindustrial localitics. S and W represent summer and winter, respectively. (Adapted with permission from ASHRAE, Handbook of Fundamentals, 1981.)

The previous calculated values were for vertical surfaces (like most windows). One can optimize the design by orienting the collection surface so that it intercepts sun rays at a better angle.

This is particularly important for solar panels placed on a roof. The southern roof slope can be chosen to face the sun rays perpendicularly.


The Rule:


A well functioning solar house needs to perform the following three functions simultaneously:

1. Capture the necessary solar energy,
2. Store heat during day for continued use through the night,
3. Distribute the heat effectively through the various rooms.

The set of these three functions is called Direct Solar Gain.

The preceding slides dealt with 1 .
The next couple of slides give an advanced glimpse of 2 . and 3 .

To store solar energy for later use:

Have a concrete slab as floor to absorb the heat and radiate it back at a later time. This is called using a "thermal mass".


Calculating the necessary thermal mass will be the subject of a subsequent lecture.
(http://solar.steinbergs.us/solar.html)

Some alternative storage methods:

masonry walls or stone/brick fireplaces

water drums...
... or why not a pool?


Distribution of heat by unforced ventilation (warm air rises and cold air sinks under buoyancy forces - the "chimney effect") is tricky business.

Effective designs provide for adequate passageways and exhaust openings.

Estimating the airflow and temperatures at various points in the structure is best accomplished by computer simulations.

(http://enertia.com/Science/)

