

A New Technique for Calculating Overall End State Probabilities for a Multiple Event Tree Probabilistic Risk Assessment

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Abstract

One of the goals for developing a Probabilistic Risk Assessment (PRA) is to estimate overall system risk by placing different types of risk into categories. However, for large complex systems it becomes difficult to get an exact answer due to the vast number of combinations that exist. Therefore, many conservative assumptions and/or approaches are used to solve the problem. Moreover, these approaches have limitations for most aerospace applications. The method described herein provides a solution that reduces the number of assumptions that have to be made and provides a more correct answer. This is done by introducing two new concepts called Pairwise Concatenation and Trumping.

Introduction

Probabilistic Risk Assessment (PRA) techniques have been used in the nuclear industry for many years and have been gaining popularity in the aerospace industry since the Challenger failure in 1986. Daniel S. Goldin, administrator of the National Aeronautics and Space Administration (NASA) from 1992 to 2001, mandated the use of formal risk management processes and technologies, such as PRA, in 2000 [1].

The objective of PRA is to support the decision-making process through identifying the contribution of causal factors to overall system risk. As part of the PRA process, the outcomes from various scenarios resulting from a sequence of events are categorized into end states. However, calculation of failure probabilities becomes increasingly difficult as the number of outcomes and the number of event trees increase. Numerical solutions can be obtained efficiently only by making a number of conservative assumptions and using approximation techniques. For modeling large complex systems such as engines, the limitations of these methods can lead to problems. Realistic estimates of end-state failure probabilities cannot be obtained.

A new methodology was developed to provide better estimates of end state probabilities while making fewer assumptions. This is achieved by introducing two new concepts called Pairwise Concatenation and Trumping.

Background

There are several ways to do a PRA. The most popular and perhaps the easiest method to understand is the small event tree approach (Refer to Figure 1 for the following discussion). Event trees consist of an initiating event (IE), pivotal events (PE), and end states (ES). Initiating events are the events that initiate a scenario, such as "Turbine Blade Fracture." End states are the outcomes of the scenario put in motion by the IE, such as "Loss of Crew/Vehicle." Pivotal events are binary events that either occur or do not occur and in doing so create combinations that form a path (sequence) from the IE to ES. Probabilities are computed for each ES within the event trees and compiled to get an overall estimate of the probability of occurrence for each ES.

For traditional widely used approaches, flaws exist in the method used to assign overall ES probabilities. A popular method is the use of minimal cutsets (mincuts) [2] to compute ES probabilities and simply sum the probabilities for like ES across multiple trees to calculate the overall ES probability. This creates two problems: 1) complement probabilities are ignored and 2) the practice of summing ES probabilities leads to inflated results. Since failure probabilities used in PRAs are often small, summing the ES probabilities to arrive at an overall ES probability is typically believed to have a negligible effect on the end result. Since the mincut analysis methodology is based on the small probability assumption described above, it breaks down when large probabilities (e.g., $p > 0.1$) are used and/or when many event trees are modeled. Example 2 demonstrates a small event tree PRA solved with the mincut method.

IE1	A	B	Sequence	Endstate
ET1 - Event Tree 1				

IE2	C	D	Sequence	Endstate
ET2 - Event Tree 2				

IE3	E	F	Sequence	Endstate
ET3 - Event Tree 3				

Figure 1. Example Event Trees

Example 1

Problem

Suppose a three-event tree PRA was performed as shown in Figure 1. Let the probability for the IE1 = 0.2, IE2 = 0.6, and IE3 = 0.4. Let the mean failure probabilities for A, B, C, D, E, and F be 0.3, 0.2, 0.8, 0.5, 0.6, 0.3, respectively. Using the mincut method show a) the formula and value for each sequence, b) the formula and value for each end state (ES) within the ET, and c) the formula and value for the each overall end state.

Solution

- a) The ES probabilities for each sequence in Event Tree 1 (ET1), Event Tree 2 (ET2), and Event Tree 3 (ET3) are computed using the mincut method as follows:

	Sequence	Endstate	Mincut Formula	Mincut Value
ET 1	1	MS	N/A	
	2	SD	IE1*B	0.04
	3	SD	IE1*A	0.06
	4	CT	IE1*A*B	0.012
ET 2	1	MS	N/A	
	2	SD	IE2*C	0.48
	3	CT	IE2*C*D	0.24
ET 3	1	MS	N/A	
	2	SD	IE3*E	0.24
	3	CT	IE3*E*F	0.072

- b) The formula for each ES within a tree follows the sequence unless more than one sequence leads to the same ES, such as in ET1. The formula for the SD ES for ET1 is therefore computed as follows:

$$SD_{ET1} = IE1 * [1 - (1-B) * (1-A)] = 0.2 * [1 - (1-0.3) * (1-0.2)] = 0.088$$

- c) To calculate the overall ES probability, simply sum the ES probabilities from each tree. This gives:

$$SD = IE1 * [1 - (1-B) * (1-A)] + IE2 * C + IE3 * E = 0.088 + 0.48 + 0.24 = 0.808$$

$$CT = IE1 * A * B + IE2 * C * D + IE3 * E * F = 0.012 + 0.24 + 0.072 = 0.324.$$

Observations

No probabilities are calculated for sequence 1 of each tree, since sequence 1 does not have cutsets. Complement probabilities are also ignored in the calculation of the sequences. Finally, if large probabilities are used or if many ETs are used it is possible to get probabilities greater than one!

A more correct method is to do the calculations using all possible combinations of all the end states within the event trees contained in the PRA. However, two obstacles are encountered when using this exhaustive method. First, the number of combinations in a typical PRA usually exceeds the power of most computers or would take an inordinate amount of time. Secondly, another problem exists where sharing of the probability space for a particular combination occurs. In aerospace (more specifically space) applications, the only available estimates of event probabilities are often conservative due to lack of information. However, reliability goals are typically minute, e.g. 0.9995 or 0.999995. The tendency toward conservatism in combination with the conventional analysis method hinders efforts to provide realistic and representative estimates of the overall probabilities. Since efforts to remove conservatism are largely contingent on the quantity and quality of data available, improving the computational methods will improve the quality of the final estimates. Therefore, an alternative algorithm must be used.

The first obstacle is the sheer magnitude of considering all combinations. For a handful of event trees, this poses no problem because the number of combinations is relatively small. In general, the number of combinations can be calculated as $N_C = (N_{ES})^{N_{ET}}$, where N_C represents the total number of combinations, N_{ES} , the total number of end states within the PRA, and N_{ET} , the number of event trees. Tables 1 and 2 show all possible combinations produced for examples with two and three event trees with three end states. The number of combinations is generalized for n event trees each with three end states in Table 3.

Table 1. All possible combinations for 2 Event Trees with 3 End states.

Combination	Event Tree 1	Event Tree 2
1	Mission Success	Mission Success
2	Mission Success	Shutdown
3	Mission Success	Catastrophic
4	Shutdown	Mission Success
5	Shutdown	Shutdown
6	Shutdown	Catastrophic
7	Catastrophic	Mission Success
8	Catastrophic	Shutdown
9	Catastrophic	Catastrophic

Table 2. All possible combinations for 3 Event Trees with 3 End states.

Combination	Event Tree 1	Event Tree 2	Event Tree 3
1	Mission Success	Mission Success	Mission Success
2	Mission Success	Mission Success	Shutdown
3	Mission Success	Mission Success	Catastrophic
4	Mission Success	Shutdown	Mission Success
5	Mission Success	Shutdown	Shutdown
6	Mission Success	Shutdown	Catastrophic
7	Mission Success	Catastrophic	Mission Success
8	Mission Success	Catastrophic	Shutdown
9	Mission Success	Catastrophic	Catastrophic
10	Shutdown	Mission Success	Mission Success
11	Shutdown	Mission Success	Shutdown
12	Shutdown	Mission Success	Catastrophic
13	Shutdown	Shutdown	Mission Success
14	Shutdown	Shutdown	Shutdown
15	Shutdown	Shutdown	Catastrophic
16	Shutdown	Catastrophic	Mission Success
17	Shutdown	Catastrophic	Shutdown
18	Shutdown	Catastrophic	Catastrophic
19	Catastrophic	Mission Success	Mission Success
20	Catastrophic	Mission Success	Shutdown
21	Catastrophic	Mission Success	Catastrophic
22	Catastrophic	Shutdown	Mission Success
23	Catastrophic	Shutdown	Shutdown
24	Catastrophic	Shutdown	Catastrophic
25	Catastrophic	Catastrophic	Mission Success
26	Catastrophic	Catastrophic	Shutdown
27	Catastrophic	Catastrophic	Catastrophic

Table 3. Pattern of combinations for a 3 End state PRA.

Event Trees	Mission Success	Shutdown	Catastrophic	Catastrophic/Shutdown	Total
1	1^1	$2^1 - 1^1$	$2^1 - 1^1$	$(3^1 - 2^1) - (2^1 - 1^1)$	3^1
2	1^2	$2^2 - 1^2$	$2^2 - 1^2$	$(3^2 - 2^2) - (2^2 - 1^2)$	3^2
3	1^3	$2^3 - 1^3$	$2^3 - 1^3$	$(3^3 - 2^3) - (2^3 - 1^3)$	3^3
n	1^n	$2^n - 1^n$	$2^n - 1^n$	$(3^n - 2^n) - (2^n - 1^n)$	3^n

Calculating probabilities resulting from this vast number of combinations can exceed the power of the typical computer. For instance, if one models three end states with 500 event trees, the total number of combinations would be $3^{500} = 3.636 \times 10^{238}$. Not only does the magnitude of combinations pose computational problems, but also the basic definition of probability states that the sum of all probabilities must equal one. The sum of all probabilities, one, must be divided into the total number of combinations. This division of the total probability may result in the truncation of probabilities at the combination level. Thus when the combinations are summed it leads to a value greater than one. Along with the necessary conservatism associated with estimates, a breakdown of this basic rule cannot be avoided.

New Method

An algorithm was developed that calculates overall end state probabilities without requiring that all possible combinations be evaluated at once. Some changes must first be made to the conventional framework as described above, through defining a global initiating event and taking the complement probabilities into account when computing sequence probabilities. Then, once each event tree's total probability sums to 1.0, exact minimum and maximum bounds for the overall end state probabilities can be computed through the use of Trumping and Pairwise Concatenation techniques.

Global IE and Complement Probabilities

In order to use the new technique, two modifications must be made to the event trees of the PRA. First, a global initiating event must be added to each event tree with an assigned probability of 1, as shown in Figure 2. Intuitively, this makes sense because the probability of occurrence for an initiating event is contingent upon the event having the opportunity to occur. For instance, in a dynamic system such as an engine, the system can fail only after it has been initiated. Examples of global initiating events include Ignition Command Given, Switch Toggled, Trigger Pulled, etc. This global initiating event serves to tie the event trees together into an integrated group.

The second part of the modification is to solve the sequences within each tree, correctly accounting for the complement probabilities of the top events. This method is sometimes referred to as “split-fraction” quantification of sequences [3]. Once the two modifications are performed as described above, the end state probabilities for each event tree will sum to one. Use of the split-fraction method in combination with the definition of a global initiating event allows all probability for the end states contained in an event tree to be taken into account. Example 2 shows how sequence probabilities are calculated after applying these modifications to Example 1.

GLOBAL-IE	IE1	A	B	Sequence	Endstate
				1	MS
				2	MS
				3	SD
				4	SD
				5	CT
ET1 - Event Tree 1					

GLOBAL-IE	IE2	C	D	Sequence	Endstate
				1	MS
				2	MS
				3	SD
				4	CT
ET2 - Event Tree 2					

GLOBAL-IE	IE3	E	F	Sequence	Endstate
				1	MS
				2	MS
				3	SD
				4	CT
ET3 - Event Tree 3					

Figure 2. Example Event Trees with the Global-IE modification.

Example 2

Problem

Using the values from Example 1 and the modified event trees shown in Figure 2, show a) the formula and value for each sequence and b) the formula and value for each end state within the ET using the “split fractions” method.

Solution

- a) The end state probabilities for each sequence in Event Tree 1 (ET1), Event Tree 2 (ET2), and Event Tree (ET3) are computed using the split fractions method as follows:

	Sequence	Endstate	Split Fractions Formula	Split Fractions Value
ET1	1	MS	1-IE1	0.8
	2	MS	IE1*(1-A)*(1-B)	0.112
	3	SD	IE1*(1-A)*B	0.028
	4	SD	IE1*A*(1-B)	0.048
	5	CT	IE1*A*B	0.012
ET2	1	MS	1-IE2	0.4
	2	MS	IE2*(1-C)	0.12
	3	SD	IE2*C*(1-D)	0.24
	4	CT	IE2*C*D	0.24
ET3	1	MS	1-IE3	0.6
	2	MS	IE3*(1-E)	0.16
	3	SD	IE3*E*(1-F)	0.168
	4	CT	IE3*E*F	0.072

- b) Since the end states are considered independent and mutually exclusive, calculations of the end state probabilities within each event tree are done using simple addition, as shown.

Event Tree 1

$$MS_{ET1} = [1-IE1] + [IE1*(1-A)*(1-B)] = 0.8 + 0.112 = 0.912$$

$$SD_{ET1} = [IE1*(1-A)*B] + [IE1*A*(1-B)] = 0.028 + 0.048 = 0.076$$

$$CT_{ET1} = IE1*A*B = 0.012$$

Event Tree 2

$$MS_{ET2} = [1-IE2] + [IE2*(1-C)] = 0.4 + 0.12 = 0.52$$

$$SD_{ET2} = IE2*C*(1-D) = 0.24$$

$$CT_{ET2} = IE2*C*D = 0.24$$

Event Tree 3

$$MS_{ET3} = [1-IE3] + [IE3*(1-E)] = 0.6 + 0.16 = 0.76$$

$$SD_{ET3} = IE3*E*(1-F) = 0.168$$

$$CT_{ET3} = IE3*E*F = 0.072$$

Trumping

Once the sequence probabilities within each event tree sum to one, the overall end state probabilities can be computed. Two items, which impact the end result, must now be considered.

1. How each particular combination of end states is treated in the calculations.
2. The probability that a particular combination of end states occurs.

As discussed earlier, all possible combinations of end states across the event trees must be considered in estimating the overall end state probabilities because of an inherent precedence ordering within the PRA. For example, the first combination represented in Table 2 occurs when none of the initiating events modeled occur: i.e. ET1, ET2, and ET3 all result in an outcome of mission success. In the second combination considered ET1 and ET2 both result in mission success while ET3 results in a shutdown. This combination would result in a shutdown. Problems arise when dealing with combinations that contain both shutdowns (SD) and catastrophic (CT), where no clear hierarchical precedence relationship is exhibited. Because of the physical nature of the system, a shutdown and catastrophic failure cannot occur simultaneously. These are mutually exclusive events and the occurrence of one precludes the occurrence of the other. Neither of these two end states dominates the other, as both do when interacting with the mission success (MS) end state. The method used to assess the behavior of the system overall is called “trumping”. This rule defines the system-level end state on an event tree basis.

To better illustrate the motivation behind trumping, first consider a simple two-event tree PRA with three end states. In this simple example, only one combination yields mission success; that is, both trees must have MS as the outcome (See Figure 3). This is because mission success is universally passive. This simply means that an outcome of mission success for any event tree is dominated absolutely by the shutdown or catastrophic failure of another event tree. For example, the outcome for a ten-event tree PRA would be SD if nine trees result in MS and one results in SD. This poses no problem due to the relationship between MS and SD. However, when a combination of end states that are not passive in regards to each other is produced, a

choice between them must be made. In Figure 3, a combination exists where ET1 produces shutdown while ET2 produces catastrophic and vice versa (highlighted in yellow). In these cases either shutdown or catastrophic must occur prior to the other; they cannot both occur. Therefore, the idea of trumping was used to identify which end state takes precedence in these situations. The conservative approach allows catastrophic failure to “trump” shutdown, resulting in a worst case upper bound on the calculated end state probability. A lower bound can be calculated similarly by allowing shutdown to trump catastrophic failure. This trumping technique is not a function of the relative importance of end states; it is merely a device for bounding the probability.

<i>Event Tree 1</i> <i>Event Tree 2</i>	Mission Success	Shutdown	Catastrophic
Mission Success	Mission Success	Shutdown	Catastrophic
Shutdown	Shutdown	Shutdown	Catastrophic/Shutdown
Catastrophic	Catastrophic	Catastrophic/Shutdown	Catastrophic

Figure 3. Combination Matrix for two Event Trees with three End states.

The next step is to compute the overall end state probabilities for the PRA after using the selected trumping rule to determine the overall end states for all possible combinations. The exhaustive combinatorial approach is first shown to demonstrate the method.

In order to demonstrate the exhaustive combinatorial approach Table 4 is generated using the values obtained for the end states in Example 2. The sum of all combination probabilities should be one, as shown in Table 4. Overall end-state probabilities can be calculated by summing the combination probabilities associated with each particular end state. For the example, the overall end state probabilities allowing catastrophic to trump are 0.360, 0.336, and 0.303 for MS, SD, and CT, respectively. If Table 4 were setup to allow shutdown to

trump, the corresponding overall end state probabilities are 0.360, 0.416, and 0.224. Comparing these results to the values obtained in Example 1 using the Mincut method, one can see how much more conservative the Mincut method is.

Table 4. End state and probability determination

ET1		ET2		ET3		END STATE	Probability
MS	0.91	MS	0.52	MS	0.76	MS	0.360
MS	0.91	MS	0.52	SD	0.17	SD	0.080
MS	0.91	MS	0.52	CT	0.07	CT	0.034
MS	0.91	SD	0.24	MS	0.76	SD	0.166
MS	0.91	SD	0.24	SD	0.17	SD	0.037
MS	0.91	SD	0.24	CT	0.07	CT	0.016
MS	0.91	CT	0.24	MS	0.76	CT	0.166
MS	0.91	CT	0.24	SD	0.17	CT	0.037
MS	0.91	CT	0.24	CT	0.07	CT	0.016
SD	0.08	MS	0.52	MS	0.76	SD	0.030
SD	0.08	MS	0.52	SD	0.17	SD	0.007
SD	0.08	MS	0.52	CT	0.07	CT	0.003
SD	0.08	SD	0.24	MS	0.76	SD	0.014
SD	0.08	SD	0.24	SD	0.17	SD	0.003
SD	0.08	SD	0.24	CT	0.07	CT	0.001
SD	0.08	CT	0.24	MS	0.76	CT	0.014
SD	0.08	CT	0.24	SD	0.17	CT	0.003
SD	0.08	CT	0.24	CT	0.07	CT	0.001
CT	0.01	MS	0.52	MS	0.76	CT	0.005
CT	0.01	MS	0.52	SD	0.17	CT	0.001
CT	0.01	MS	0.52	CT	0.07	CT	0.000
CT	0.01	SD	0.24	MS	0.76	CT	0.002
CT	0.01	SD	0.24	SD	0.17	CT	0.000
CT	0.01	SD	0.24	CT	0.07	CT	0.000
CT	0.01	CT	0.24	MS	0.76	CT	0.002
CT	0.01	CT	0.24	SD	0.17	CT	0.000
CT	0.01	CT	0.24	CT	0.07	CT	0.000
Total							1.000

Pairwise Concatenation

The exhaustive combinatorial approach may be used when a small number of event trees are being modeled. However, this approach is typically not feasible because of the number of combinations and the use of small probabilities. An alternative method, Pairwise Concatenation, was developed to streamline the computational aspect of this problem.

The new process for computing overall end state probabilities is conceptually simple. Two event trees are solved for the pairwise end state probabilities, creating a new event tree called the Transitional Event Tree (TET). This first TET is then combined with the third ET to

obtain another TET; the process is then repeated until all the trees are combined and a single TET has resulted. This last TET is the final result; results are mathematically equivalent to those obtained through the exhaustive combinatorial approach. This method also drastically reduces the number of operations necessary for computation. A graphical representation of Pairwise Concatenation is shown in Figures 4 and 5. Example 3 computes the overall end state probabilities using the new methodology.

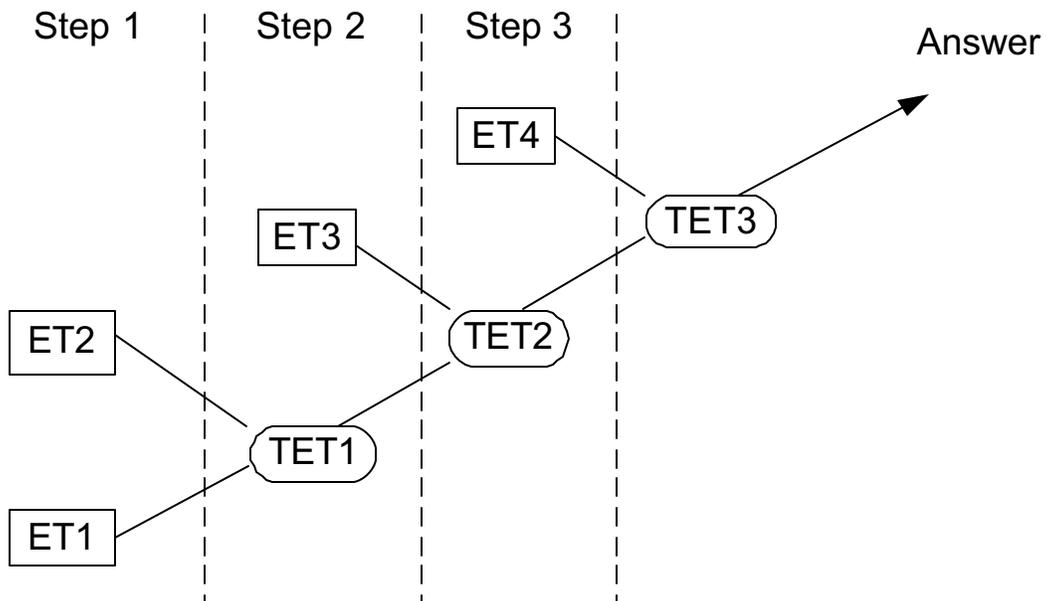


Figure 4. The Pairwise Concatenation Process.

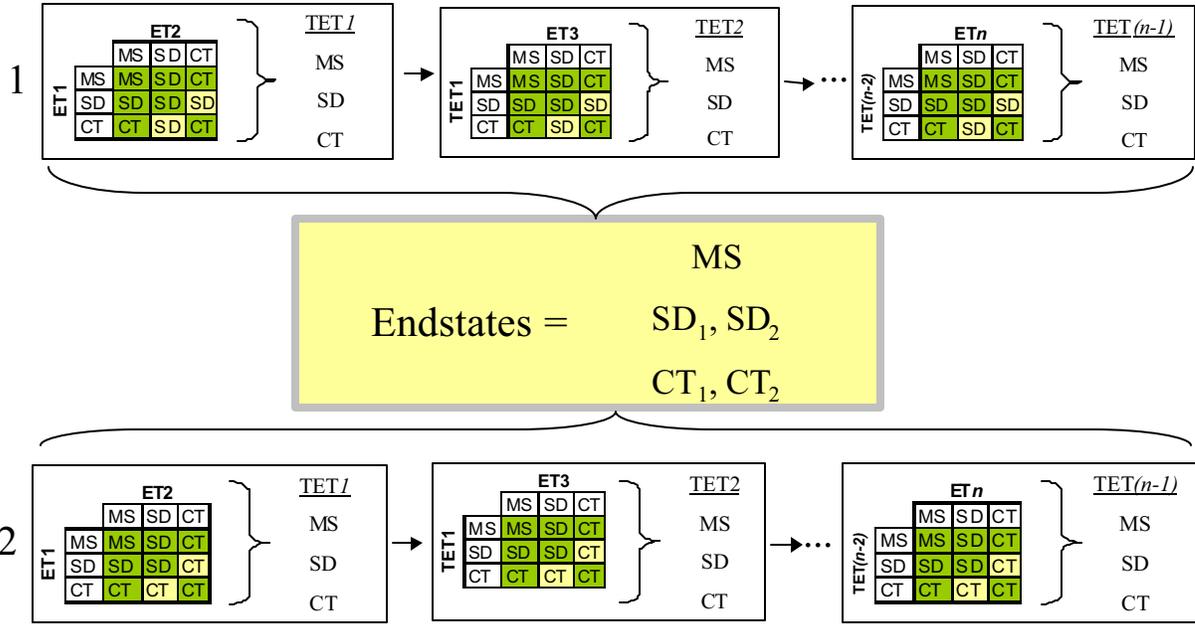


Figure 5. The Pairwise Concatenation Process with Trumping of End states.

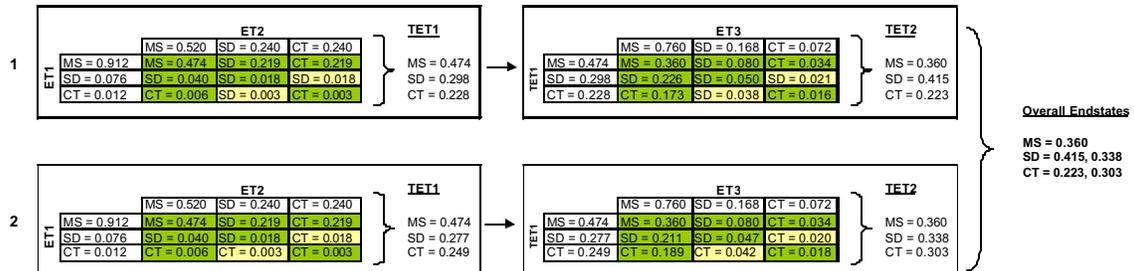
Example 3

Problem

Using the modified event trees shown in Figure 2 and the values computed for the end states in part B of Example 2, solve for the overall endstates using pairwise concatenation and trumping, where Shutdown (SD) trumps Catastrophic (CT) and vice-versa.

Solution

As illustrated below path 1 lets SD trump CT and path two lets CT trump SD. ET1 combines with ET2 to yield TET1. TET1 then combines with ET3 and they yield TET2. Path one is anti-conservative and path two is the conservative path. The overall end states are shown as the combination of the two paths.



Observations

The sums of all the combinations equal one. There is only one combination that leads to Mission Success. The true answer for SD and CT lies between the anti-conservative/conservative range. In a real PRA, the probabilities will most likely be small thus the range would not be as large as shown here.

Summary

The new technique described here for solving overall end state probabilities may be summarized in three steps: 1) Add a global initiating event to each event tree, 2) Solve for the end states within each tree correctly using complement probabilities (split-fractions method), and 3) use pairwise concatenation in conjunction with trumping to solve the combinations and arrive at the final estimates of overall end state probabilities. This proposed method for calculating overall end state probabilities offers a viable alternative to the current commonly used approach. For PRAs with a potentially high number of end state categories and large numbers of event trees, this approach offers a computationally succinct and pragmatic way of obtaining more precise estimates of failure probabilities for the outcomes of interest. Performing pairwise concatenation in conjunction with trumping yields identical results as using the exhaustive combinatorial approach with trumping.

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