Shielding of Gamma Radiation

George E. Chabot, Jr., PhD, CHP

Introduction

In the discussion that follows, we assume that gamma rays are the radiation of interest. The principles discussed also apply to monoenergetic x rays; in many cases of concern in x-ray shielding, however, the photons are produced in the bremsstrahlung process that yields a wide and continuous distribution of photon energies. Specialized shielding approaches have been developed to carry out shielding estimations for such x rays (e.g., NCRP 2003, NCRP 2004), and these will not be explicitly discussed.

Our emphasis here will be on the development of an analytical expression based on what is often referred to as the point kernel method. We will obtain an expression for the shielded dose rate from a point isotropic source and show how that can be used to obtain deterministic solutions for other source geometries. For fixed input parameter values, deterministic methods always produce the same answer. Deterministic solutions are, by their nature, single-valued, fixed solutions, not subject to random kinds of fluctuations associated with other approaches (which are usually being called stochastic or probabilistic methods).

In the category of probabilistic methods are the well-known and very popular Monte Carlo techniques embodied in computer codes such as MCNP (Monte Carlo N-Particle transport code) and EGS4 (Electron Gamma Shower code); both codes are available from the Radiation Safety Information Computational Center at Oak Ridge. In Monte Carlo simulations of shielding problems, the code tracks the fates of individual photons as they move through the shield; decisions as to how far a photon travels before interacting, what type of interaction occurs, what direction is taken by scattered photons, etc., are all made on a probabilistic basis in which random numbers are selected and associated with specific probabilities that are used to specify the decision outcome.

Thus, if the fates of two identical photons are followed, we would expect them to be different much of the time, and it is only by investigating enough photons that we might expect the overall results to be representative of reality—e.g., that the determined dose at a receptor location will be correct. Two individuals running the same Monte Carlo code to solve the same problem will not necessarily arrive at exactly the same answers.

Deterministic methods have the advantage that the computations are very fast and relatively easy to carry out. Some disadvantages are that they are not very useful for complex source and/or shield geometries, dispersed energy sources, inhomogeneous sources and/or shields, laminated shields, and streaming-type calculations in which one might be concerned about the leakage of radiation through a shield penetration such as a conduit. Monte Carlo codes are amenable to these more complex shielding problems and have become more and more popular as high-speed computing has become available to so many people. In general, however, they do require considerably more expertise and training to use and are often much slower in reaching a solution than are the deterministic methods.

Photon interactions and secondary radiations

When gamma radiation is incident on a finite thickness of material, there exists some probability that the radiation will interact in the material and be attenuated. In some instances a photon may interact by the photoelectric effect, in which case the photon disappears after transferring all of its energy to a bound electron, which gets ejected from the atom. When the vacancy left in the shell by the removed electron gets filled by an electron dropping into it from a higher energy level, the difference in energy between the two transition states may appear as a fluorescent photon. These photons are characteristically low in energy, but some may be capable of reaching the dose point inside or outside the shielding material. The photoelectric process is favored for low-energy photons interacting in a high atomic number (Z) material.

At moderate and higher energies another process, called Compton scattering, prevails; in this process only a portion of the photon energy is transferred to an electron, and a scattered photon moves away from the interaction site, often in a direction different from that of the original photon. This scattered photon may find its way to a dose point of interest inside or outside the attenuating material.

At energies exceeding 1.022 MeV, especially in higher-Z materials, the pair production interaction process may occur. In this event the photon interacts in the Coulomb field of the nucleus, with all of its energy being transformed into mass in the form of a conventional electron and a positively charged electron (positron). Any original photon energy beyond the 1.022 MeV required to generate the mass associated with the electron-positron pair will appear as kinetic energy of the pair. After the positron has dissipated its kinetic energy it will disappear in an annihilation event with a conventional electron, in the process producing two 0.511 MeV annihilation photons that move apart in opposite directions. Occasionally, the positron may annihilate during its flight, and in this case whatever kinetic energy it has will be transferred to the annihilation photons, one or both of which may now have energy greater than 0.511 MeV. In this instance the annihilation photons do not expectedly move in opposite directions. In any case, annihilation photons may also find their way to the dose point. Pair production events may also occur in the Coulomb field of an electron, but the incident photon energy must exceed 2.044 MeV for this to occur.

Thus, any of the common gamma interaction processes may result in secondary photons that have a finite probability of reaching the dose point. The extent to which such secondary photons add to the fluence or dose at the dose point is usually described through the use of an appropriate buildup factor.

Buildup factors may refer to various quantities of interest, such as photon fluence, photon energy fluence, exposure, or dose, and the values among all are somewhat different. For most of our discussion here we shall assume that the dose or exposure buildup factor is of interest. Much of the available buildup data relates to determination of exposure or kerma in a small air volume envisioned to be located within the shielding medium of interest. These data are also suitable for evaluation of dose to water or other low-Z material of interest.

The dose buildup factor is a dimensionless quantity that represents the ratio of total dose (including the dose from secondary photons) at the dose point to primary photon dose at the same point. The primary photon dose naturally comes from original photons that have penetrated the shielding material without interacting. Magnitudes of buildup factors vary widely, ranging from a minimum of 1.0 to very large values, depending on source and shield characteristics.

Good geometry shielding situation

When a narrow parallel beam of photons passes through a relatively thin shield, and if the dose point is many beam diameters away from the exit surface of the shield, we have a situation referred to in photon shielding as good geometry. This means, simply, that virtually all of the photons arriving at the dose point will be primary photons, and the dose, D, or dose rate, at a point of interest outside the shield, is related to the unshielded dose, D_0, or dose rate, at the point by

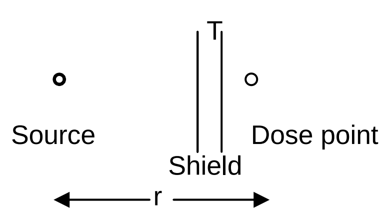
$$D = D_0 e^{-\mu x} \quad (1),$$

where μ is the linear attenuation coefficient for the photons of the energy of interest in the shield material, and x is the shield linear thickness. Values of μ are available in a variety of sources, one convenient one being the National Institute of Standards and Technology (NIST). The values at the NIST Web site are actually mass attenuation coefficients, and the values must be multiplied by the shield material mass density to obtain the respective linear attenuation coefficients.

Point isotropic source

Probably the most popular source geometry involved in many calculations is the point isotropic source. While no real source is a true point, many sources are sufficiently small in dimensions that they can be treated mathematically as point sources. In practice, if the distance from source to dose point exceeds about three times the maximum source dimension, and self-attenuation within the source volume is not a concern, the errors resulting from treating the source as a point will not exceed a few percent. The assumption that the source is isotropic means that radiation of concern is emitted uniformly in all directions throughout a 4π geometry. We shall define such a source of monoenergetic gamma radiation that emits S gamma rays per second and that is situated at a distance r (cm) from the dose point. Further, we shall assume a shield of thickness T (cm) through which the gamma radiation passes before reaching the dose point (see sketch below):

Point isotropic source shielding configuration



Unshielded dose rate

The unshielded dose rate at the dose point is given by

$$\dot{D} = \frac{kSE\mu_{en} / \rho}{4\pi r^2} \quad (2),$$

where E is the photon energy, MeV,

μ_{en}/ρ is the mass energy absorption coefficient for the material at the dose point, $cm^2\ g^{-1}$ (values also available at NIST), and

k is a collective constant to convert energy fluence rate to dose rate; if the dose rate is in gray/hour, k will have a value of 5.76×10^{-7}.

Shielded primary photon dose rate

The primary photon dose rate is attenuated exponentially, and the dose rate from primary photons, taking account of the shield, is given by

$$\dot{D} = \frac{kSE \frac{\mu_{en}}{\rho} e^{-\mu T}}{4\pi r^2} \quad (3),$$

where μ is the linear attenuation coefficient for the photons in the shield material. This expression does not account for the buildup of secondary radiation and will generally underestimate the true dose rate, especially for thick shields and when the dose point is close to the shield surface.

Shielded dose rate accounting for buildup

The added effect of the buildup is taken into account by incorporating a point isotropic source dose buildup factor, B, into equation 3:

$$\dot{D} = \frac{kSE \frac{\mu_{en}}{\rho} Be^{-\mu T}}{4\pi r^2} \quad (4).$$

The magnitude of the buildup factor depends on the photon energy, the shield material and thickness, the source and shield geometry, and the distance from the shield surface to the dose point. In most cases, dose buildup factors for point isotropic sources have been determined under the assumption that both the source and the dose point reside within an infinite volume of the shield material. As a consequence, shielded doses evaluated using such buildup factors tend to be conservative for most practical situations in which the dose point is outside the shield and not subject to backscattering from shield material behind the dose point.

Tabulated values of buildup factors for point isotropic dose may be found in a number of sources (e.g., Bureau of Radiological Health 1970, Shultis & Faw 1996). Such values are arranged according to shield material, photon energy, and shield thickness, usually expressed as the product μT, which represents the number of photon mean free paths represented by the shield thickness. Such tabulated values are useful, especially if one knows the shield thickness and wants to determine the dose rate. When one wishes to determine the shield thickness to yield a specific dose rate, equation 4 cannot be solved explicitly for T because the value of B depends on T. Solutions can be obtained by making educated guesses for the value of T, looking up the corresponding values of B, and solving for the dose rates; results can be plotted, and the correct value of T determined for the desired value of dose rate. Alternatively, we can use an analytical form of the buildup factor that can be incorporated into equation 4 and, through an iterative process using a computer or calculator, solve for the desired thickness. There are a number of algebraic expressions that have been used to represent B.

Among the most popular is an expression referred to as Taylor's form of the buildup factor, given by

$$B = A_1 e^{-\alpha_1 \mu T} + (1 - A_1) e^{-\alpha_2 \mu T} \quad (5),$$

where A_1, α_1, and α_2 are constants for a given energy and shield material. Tabulations of these parameters can be found in various engineering and shielding sources (e.g., Shultis and Faw 1996, 2000). It should be noted that there are a variety of individual values of A_1,α_1, and α_2 that will yield the correct value of B for a given energy, shield material, and shield material thickness, so different literature sources may have quite different respective parameter values. A few other analytical forms that have been used for the buildup factor are given below:

Berger's form: $B = 1 + a\mu T e^{b\mu T}$, where a and b are constants for a given energy and shield material,

Linear form: $B = 1 + k\mu T$, where k is often taken as a constant (e.g., 0.3 to 1), but actually varies significantly with shield thickness and photon energy (not often a very accurate form), and

Polynomial form: $B = 1 + \alpha\mu T + \beta(\mu T)^2 + \gamma(\mu T)^3$, where α, β, and γ are constants for a given energy and shield material.

Taylor's form has the advantage that it has only exponential terms in μT, and when it is used in an equation that expresses the shielded dose rate, the form of the ultimate solution is fundamentally the same as the solution for the primary photons alone, except that it will have twice as many terms because of the two exponential terms in the buildup factor.

When the expression for B from equation 5 is inserted into equation 4 we obtain

$$\dot{D} = \frac{kSE\frac{\mu_{en}}{\rho}[A_1 e^{-\alpha_1 \mu T} + (1 - A_1)e^{-\alpha_2 \mu T}]e^{-\mu T}}{4\pi r^2} \quad \text{(6), or}$$

$$\dot{D} = \frac{kSE\frac{\mu_{en}}{\rho}(A_1 e^{-(1+\alpha_1)\mu T} + (1 - A_1)e^{-(1+\alpha_2)\mu T})}{4\pi r^2} \quad \text{(7).}$$

Multiple photon energies

In the above expressions we have assumed a single gamma-ray energy. When a gamma-emitting radionuclide emits more than one gamma energy, the same expressions as above may be used individually for each gamma energy; the appropriate values for S, E, μ_{en}/ρ, μ, A_1, α_1, and α_2 must be used for each distinct photon energy. The total dose rate is the sum of results for the individual photons. In some instances, when photon energies are close to each other, the photons may be grouped together by using the average energy and the combined yields. A classic example of this is for ^{60}Co, which emits 1 gamma per disintegration at 1.17 MeV and 1 gamma per disintegration at 1.33 MeV. Many shielding calculations for this nuclide have been done using an energy of 1.25 MeV and a combined yield of 2 gammas per disintegration. When energies are more disparate it is often not suitable to attempt to combine them. When quite low-energy photons are emitted along with moderate yield high-energy photons, one may often neglect the low-energy photons in doing shielding calculations because they will not contribute appreciably to the shielded dose rate. Such decisions must be made with some care, however, and generally improve with experience.

Example of shielded point isotropic source calculation

Problem: Determine the soft tissue dose rate, in gray per hour (Gy h^{-1}), at the outer surface of a 2-inch thick lead shield from a 3.0 Ci source of ^{137}Cs. Assume that the source may be treated as a point isotropic source and that its effective distance from the dose point is 6.35 cm. We shall also assume that soft tissue can reasonably be simulated by water and use the mass energy absorption coefficient for water.

Solution: Following are the values of parameters that are necessary for the solution:

k = 5.76 x 10^{-7},
S = (3.0 Ci)(3.7 x 10^{10} d s^{-1} Ci^{-1})(0.85 γ d^{-1}) = 9.44 x 10^{10} γ s^{-1},
E = 0.662 MeV, (Note: The 0.662 MeV photons of concern have a yield of 0.85 per disintegration and actually come from the daughter product ^{137m}Ba, which has a short half-life and quickly achieves equilibrium with the ^{137}Cs.)
μ= 1.289 cm^{-1},
T = 5.08 cm,

μ_{en}/ρ = 0.0326 cm^2 g^{-1},
A_1 = 2.632,
α_1 = -0.0145,
α_2 = 0.136, and
r = 6.35 cm.

Inserting values into equation 7 yields

$$\dot{D} = \frac{(5.76\ x\ 10^{-7})(9.44\ x\,10^{10})(0.662)(0.0326)(2.632\,e^{-(1-0.0145)(1.289)(5.08)} - 1.632\,e^{-(1+0.136)(1.289)(5.08)})}{4\pi(6.35)^2},$$

and $\dot{D} =$ 7.38 x 10^{-3} Gy h^{-1} or 0.738 rad h^{-1}.

We can readily calculate the part of this dose rate that is due to the primary gamma rays, alone, by solving equation 3. If we do this we will obtain a dose rate of 3.32 x 10^{-3} Gy h^{-1}, about 45% of the total dose rate. Thus, for this example, the secondary photons, which would be almost all Compton scattered photons, account for more than half of the dose rate. The magnitude of the buildup factor, given by the ratio of the total dose rate to the primary photon dose rate, is 2.22. In general, as shield thickness increases, the fraction of the dose attributable to secondary photons increases. The shield in this example had a thickness of 6.55 mean free paths (μT = (1.289)(5.08)). Had we had a shield that was 20 mean free paths thick we would have found that the buildup factor was about 3.4, implying that more than 70% of the dose rate would have been from secondary photons.

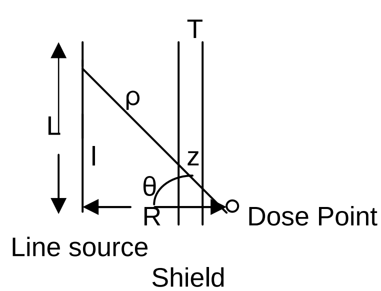
It is interesting to observe that if the above problem had dealt with a ^{137}Cs source that was shielded by water rather than by lead, but with the same number of mean free paths of material (i.e., 6.55), the buildup factor would have been larger than that calculated above by about a factor of 10; thus, the portion of the dose rate that would have been associated with the secondary photons would have been about 95%. This is because in water the photons must undergo many more scattering processes than they do in lead before they are ultimately captured and disappear in a photoelectric interaction. The reason for this is because the probability of a photoelectric event has a strong dependence on atomic number of the material, increasing as about the 4^{th} to 5^{th} power of atomic number, and lead has a much higher atomic number than water (82 for lead compared to about 7 for water). In water the photon energy must be reduced to somewhat less than 30 keV before the photoelectric and Compton interactions occur with equal probability, while in lead the analogous energy is slightly less than 600 keV.

We should recognize that to solve the above problem we could have simply looked up the value of the point isotropic source buildup factor in one of the cited compilations. Rather, we did the problem using the analytical form of the buildup factor to illustrate its use for other applications. If, for example, we had been asked to determine what shield thickness of lead would have been appropriate to yield a specified dose rate at the shield surface, we could have plugged the required dose rate into equation 6 or 7 and used either computer software or an appropriate pocket calculator equipped with a hard-wired “solve” routine to determine the necessary lead thickness. Additionally, the use of an algebraic expression for buildup allows extension of the point kernel method employed above to other source geometries.

Extended geometries

Once we have an expression for the point isotropic source, we can write reasonable expressions that will apply to other nonpoint source geometries by recognizing that any extended source geometry can be represented by an infinite number of points distributed throughout the source dimensions. The unfortunate aspect of this approach is that even for the next simplest geometry—i.e., a uniform line source—we cannot obtain a neat closed form algebraic solution to shielding problems. We can, however, write the differential equations that describe the dose rate from one generalized differential element in the source and then, by numerical integration, add up the contributions from all such elements to obtain final dose rates. We will demonstrate this through a line source application. Let us assume that we have a gamma-emitting source distributed uniformly along the length of a line. In reality many sources that have one straight line dimension much greater than any other dimensions may be treated as a line source. We shall assume a source of length, L, with the dose point opposite the end of the line source and along a line perpendicular to the source. The gamma-ray emission rate per unit length of source will be given by S_l, which has units of gammas per cm per second. The shield of uniform thickness, T, will be between the source and the dose point as shown below:

Line source shielding configuration



Let us select a small (differential) length element, dl, of the line source, located at a distance, l, from the lower end of the source at the point in the above diagram where the oblique line, ρ, from the dose point meets the source line. The angle between R and ρ we shall call θ. From the geometry shown we can specify the following:

$\rho = R \sec\theta$;
$l = R \tan\theta$ and, by differentiation, $dl = R \sec^2\theta \, d\theta$
z = radiation path length through shield = $T \sec\theta$, and
$S_l dl$ = gamma emission rate from differential source element, dl.

The shielded differential dose rate at the dose point from primary photons emitted from the differential source element may then be written:

$$d\dot{D} = \frac{kS_l dlE \frac{\mu_{en}}{\rho} e^{-\mu T \sec\theta}}{4\pi\rho^2} = \frac{kS_l R\sec^2\theta d\theta E \frac{\mu_{en}}{\rho} e^{-\mu T \sec\theta}}{4\pi R^2 \sec^2\theta} = \frac{kS_l E \frac{\mu_{en}}{\rho} e^{-\mu T \sec\theta} d\theta}{4\pi R} \quad (8).$$

To obtain the total primary photon dose rate from all differential source elements along the line source we have simply to integrate the above expression over the range of the variable, θ, i.e., from zero to the angle whose tangent is L/R:

$$\dot{D} = \frac{kS_l E \frac{\mu_{en}}{\rho}}{4\pi R} \int_0^{\tan^{-1}\frac{L}{R}} e^{-\mu T \sec\theta} d\theta \quad (9).$$

The integral is of a form usually identified as the Sievert integral or the secant integral. An exact solution is not available for this integral, but it is easily solved using available computer software or a programmable calculator that has integration capability. There are also tables available that yield acceptable approximate solutions of the integral for given values of θ and μT. One advantage to performing the above calculation for the dose rate at a point opposite the end of the line source is that if the dose point in another case is opposite some other part of the line source, the result for such case can be readily obtained by adding together the dose rates from two line sources where the dose point is opposite the end of each line source. Thus, for a situation where the dose point was on a line perpendicular to the line source, and the source length above the dose point was L_1 and the source length below the dose point was L_2, the primary photon dose rate would be

$$\dot{D} = \frac{kS_l E \frac{\mu_{en}}{\rho}}{4\pi R} [\int_0^{\tan^{-1}\frac{L_1}{R}} e^{-\mu T \sec\theta} d\theta + \int_0^{\tan^{-1}\frac{L_2}{R}} e^{-\mu T \sec\theta} d\theta] \quad (10).$$

In order to account for the added dose from buildup from a single line source, again with the dose point opposite the end of the line, we would insert Taylor's expression for B into equation 8 and then proceed as earlier to obtain

$$\dot{D} = \frac{kS_l E \frac{\mu_{en}}{\rho}}{4\pi R} (\int_0^{\tan^{-1}\frac{L}{R}} A_1 e^{-(1+\alpha_1)\mu T \sec\theta} d\theta + \int_0^{\tan^{-1}\frac{L}{R}} (1-A_1) e^{-(1+\alpha_2)\mu T \sec\theta} d\theta) \quad (11).$$

It is clear that the form of the solution with buildup is fundamentally the same as that for the primary photons except that there are two similar terms that arise from the two terms in the buildup factor, and where the parameter μ appeared in the equation for primary photons the parameters $(1+\alpha_1)\mu$ and $(1+\alpha_2)\mu$ appear in each of the respective terms in the solution when buildup is considered. For the case when the dose point is opposite some other point on the line source we would obtain

$$\dot{D} = \frac{kS_l E \frac{\mu_{en}}{\rho}}{4\pi R} \sum_{i=1}^{2} \left(\int_0^{\tan^{-1}\frac{L_i}{R}} A_1 e^{-(1+\alpha_1)\mu T \sec\theta} d\theta + \int_0^{\tan^{-1}\frac{L_i}{R}} (1-A_1) e^{-(1+\alpha_2)\mu T \sec\theta} d\theta \right) \quad (12).$$

The summation from i = 1 to i = 2 accounts for the two line segments, L_1 and L_2. Again, the solution is of the expected form, now with four terms, two for the dose rate contribution from each line segment.

Example of shielded line source calculation

We will perform a calculation again for 3 Ci of ^{137}Cs with the same two inches of lead shielding, but in this case we will assume the activity is distributed uniformly along a line 1 meter in length and that the dose point is at a perpendicular distance of R = 6.35 cm from the source and opposite the center of the source. All the attenuation and buildup parameter values are the same as those used in the point source calculation. Since the dose point is opposite the source center we will use equation 11, with L = 50 cm, and multiply the result by two (or use equation 12 with $L_1 = L_2$). The value of S_l is obtained by dividing the total gamma emission rate by the length of the source, which yields 9.44 x 10^{10} γ s^{-1}/100 cm = 9.44 x 10^8 γ s^{-1} cm^{-1}. Note that the upper limit of the angle θ is the arctangent of L/R = $\tan^{-1}$(50/6.35) = 1.44 radians.

$$\dot{D} = \frac{(2)(5.76x10^{-7})(9.44x10^{8})(0.662)(0.0326)}{4\pi(6.35)} \left[\int_0^{1.44} 2.632 e^{-(1-0.0145)(1.289)(5.08)\sec\theta} d\theta - \int_0^{1.44} 1.632 e^{-(1+0.136)(1.289)(5.08)\sec\theta} d\theta \right]$$

The first integral yields a numerical integration value of 1.884 x 10^{-3}, the second integral value is 4.099 x 10^{-4}, and the terms in front of the integrals combine to a value of 0.294, the final dose rate then being 4.33 x 10^{-4} Gy h^{-1}. This dose rate is almost 20 times less than that from the equivalent activity point source analyzed above; this is expected because, although the perpendicular distance from the dose point to the line source is the same distance used for the point source calculation, most of the source activity of the line source is considerably farther away from the dose point than in the case of the point source. In addition the shield is more effective for the line source because much of the gamma radiation travels oblique paths through the shield, thus increasing the distance traveled through the shield material. We might also note that if we solved for the primary photon dose rate alone, we would have obtained 1.90 x 10^{-4} Gy h^{-1}. If we divide this number into the total dose rate obtained above we get 2.28, which conceptually represents the average value of the dose buildup factor for this shielding situation. This value is slightly larger than the value of 2.22 obtained for the point source because of the effect of photons traversing longer path lengths through the shield in the case of the line source.

It is a fairly easy matter to extend the analyses to other regular geometries, including area sources and volume sources, but these cases are somewhat beyond the scope of intentions here. In such cases we proceed in a fashion similar to what we did for the line source except that we must take proper account of the different geometries. For area sources the differential source element will be an area element, and the source strength will be defined in terms of gamma emission rate per unit area; for volume sources we must specify a differential source volume element and a gamma emission rate per unit volume. In the case of a volume source we may also have to account for attenuation within the source volume, the volume itself becoming a shield for the radiation.

References

Bureau of Radiological Health. Radiological health handbook. Rockville, Maryland: US Department of Health Education and Welfare, Public Health Service, Food and Drug Administration; 1970.

National Council on Radiation Protection and Measurements. Radiation protection for particle accelerator facilities. Bethesda, MD: NCRP; Report No. 144; 2003.

National Council on Radiation Protection and Measurements. Structural shielding design for medical x-ray imaging facilities. Bethesda, MD: NCRP; Report No. 147; 2004.

Shultis JK, Faw RE. Radiation shielding. Upper Saddle River, New Jersey: Prentice Hall; 1996: republished in 2000 under same title by the American Nuclear Society.