Radiation dosage and its Biological effects

I. RadioActivity: the number of radioactive decay per second of a sample.

Since $dN/dt = -\lambda N$, the activity is $A = \lambda N$. 1 g Co60 ($\tau \sim 5.27$y) contains about 50 Ci

1 Bq (becquerel)	1 disintegration/s
1 Ci (curie)	3.7×10^{10} decays/s ~ the activity of 1g of ^{226}Ra.

II. Unit of **E**xposure and (**A**bsorbed dose) and **D**ose equivalent

1 X-unit	1 C/kg of dry air
1 R (Roentgen)	2.58×10^{-4} C/kg of dry air
1 **rad** (radiation absorbed dose)	1 erg/g=0.01 J/kg
1**Gy** (gray) =100 rad	1 J/kg
1 DE (dose effective)	(absorbed dose)×**RBE** (QF)
1 **rem** (rad equivalent in man)	(absorbed dose in rad) ×RBE (QF)
1 **Sv** (sievert) [GyE/CGE]	(absorbed dose in Gy) ×RBE (QF)

CGE=Cobalt Gray Equivalent; organ at risk (OAR); gross tumor volume (GTV); CTV (clinical target volume) = GTV + 5-10 mm; planned treatment volume PTV = CTV + 5~10 mm; dose volume histograms (DVH)

Background radiation is about 130 mrem/y (1.3 mSv/y), or 0.15 μ(micro)Sv/h; US regulation is 5mSv/y, radiation worker 50 mSv/y. In ICRP Publication 62, a representative value of 1.8 mSv (180 mrem) effective dose is given for a head CT.

III. Stopping power ($-dE/dx$) is the energy lost by a charged particle in a medium.

IV. LET is the energy absorbed in the target.

There are evidences that radiation in large doses causes harms to human body (see ICRP 109). However, there is no "acceptable threshold" dose that the cancer risk begins to increase. Some environmental radiation doses are listed below:

Background radiation	3.5 mSv (cosmic ray 1 mSv, other 2 mSv)
Smoking for 1 year	13 mSv
Medical X-ray	0.1 mSv
Cross continent plane trip	0.06 mSv
Radiation worker limit	5 mSv
CT scan of head (body weighted)	1.8 mSv
Whole body CT	12-25 mSv
mammography	0.45-0.84 mSv
US limit of medical radiation	1 mSv
US limit for nuclear worker	**50 mSv**

20 million adults and 1 million children may be irradiated unnecessarily each year in the US.

Apollo 14 astronaut	11.4 mSv
Astronaut in skylab4 (87 days)	178 mSv
Radiation sickness symptons	500 mSv
Round trip to Mars (2.5 y)	1300 mSv
Catastrophic exposure	5000 mSv (50% death)

A simple Dosage Calculation Example

1. A 5-MeV α particle is **absorbed** by 1 gram of water, estimate the dosage in rad and rem.

$$\frac{5\text{MeV}}{1\text{ g}}\frac{1.6\times10^{-13}\text{ J}}{1\text{ MeV}}\frac{10^{7}\text{ erg}}{1\text{ J}}\frac{1\text{ rad}}{100\text{ erg/ g}} = 8.0\times10^{-8}\text{ rad}$$

The RBE (Q factor) is 10 for α particle, and thus the dose is 8E-7 rem or 8E-9Sv. If the α particle is absorbed by a of 10^{-9} g cell, then the dose is 10^{9} times higher (0.8 Gy, 8 Sv), exceeded lethal dose for most living beings.

2. Proton at 250 MeV are used for radiation therapy with a treatment volume of 1 kg. Assuming 70% efficiency in reaching the PTV. What is the number of protons per second needed for the dosage of 2 Grays in 2 minutes?

$$\frac{250\text{MeV}}{1\text{ kg}}\frac{1.6\times10^{-13}\text{ J}}{1\text{ MeV}}\,N\times120s\times70\% = 2\text{ J/kg}$$

$$N = 6\times10^{8}\ \text{particles/second}$$

History of Radiation dosage measurements

- 1908: Paul Ulrich Villard used ionization to quantify the radiation
- 1925, international congress of radiology (ICR) meeting (London)
- 1925: ICRU (international commission on radiological units and measurements
- 1928: ICR formed an "International X-ray and Radium Committee," which was to then evolve to become the ICRP (International Commission on Radiological Protection) in 1950.
- 1950: ICR meeting in London, the absorbed dose was defined to describe the amount of energy absorbed per unit mass of an irradiated medium at a point in that medium. The CGS unit is erg / gram.
- 1953 in Copenhagen, the **rad** was defined as the unit of absorbed dose The rad was an acronym for radiation absorbed dose, where 1 rad = 100 erg/g.
- 1970s, ICRU Report 33 (1980) chose the SI units for the absorbed dose. The **kerma** unit was defined as the "kinetic energy released per unit mass" by indirectly-ionizing radiation (photons and neutrons). The unit for kerma is gray, defined as 1 Gy = 1 J/kg. 1 rad= 1 cGy.
- 1956: IAEA was formed under the United Nations!

Kerma vs the absorption dose

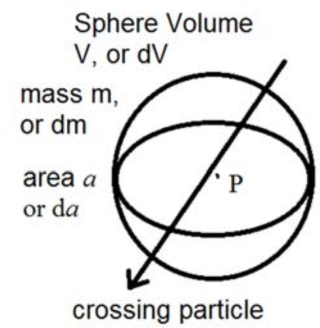
Example: A 10-MeV photon penetrates a 100-g mass, and a pair production leads to a positron and an electron of 4.5 MeV each. Three bremstrahlung photons of 1.6, 1.4 and 2 MeV are produced and escaped from the mass before they interact. The positron after losing all its kinetic energy produces two photons at 0.51 MeV (assuming absorbed). What are the kerma and absorbed dose?

Kerma: $$K = \frac{2\times 4.5\text{MeV}\times 1.6\times 10^{-13}\,\text{J/MeV}}{0.1\text{kg}} = 1.4\times 10^{-11}\,\text{Gy}$$

Dose $$D = \frac{[2\times 4.5-(1.6+1.4+2)]\text{MeV}\times 1.6\times 10^{-13}\,\text{J/MeV}}{0.1\text{kg}} = 6.4\times 10^{-12}\,\text{Gy}$$

Description of ionizing radiation fields

To describe radiation field at a point P, one needs to define non-zero volume around it. The radiation is divided into stochastic or deterministic (non-stochastic) physical quantities



1. Fluence: $\Phi = \frac{dN}{da}$
2. Flux density (Fluence rate): $\varphi = \frac{d\Phi}{dt} = \frac{d}{dt}\frac{dN}{da}$
3. Energy Fluence: $\Psi = \frac{dEN}{da}$
4. Energy Flux density (Energy Fluence rate): $\psi = \frac{d\Psi}{dt} = \frac{d}{dt}\frac{dEN}{da}$

- Photon beam attenuation: $I(t) = I_0\, e^{-\mu t}$, where I(t) is intensity transmitted by an absorber of thickness t, I_0 is the incident intensity, and μ(E, Z) the attenuation co-efficient

Relationship between μ, mass attenuation coefficient, μ_m , atomic attenuation coefficient, ${}_a\mu$, electronic attenuation coefficient ${}_e\mu$ is the following

$$\mu = \rho\mu_m = \frac{\rho N_A}{A}\,{}_a\mu = \frac{\rho N_A Z}{A}\,{}_e\mu$$

The energy transfer (E_{tr}) coefficient (μ_{tr}), is related to the energy absorption (E_{ab}) coefficient μ_{ab} or μ_{en} by

$$\mu_{tr} = \mu\frac{\bar{E}_{tr}}{h\nu} \qquad \mu_{ab} = \mu\frac{\bar{E}_{ab}}{h\nu} \qquad \mu_{ab} = \mu_{tr}(1-g)$$

Here "g" is the fraction of the energy transfer lost through radiative process.

Exposure

At low energy, the radiation may induce electron excitation to jump from lower shells to upper shells, and the excitation may break molecular bonds and cause damage to living cells. At high energy, the radiation can free electrons from its atomic orbit and produce ionizations, this is called **ionizing radiation**. Otherwise, it is called non-ionizing radiation.

The interaction of gamma and X-rays in air, the **exposure** measures the electric charge (positive or negative) produced by electromagnetic radiation in a unit mass of air, at the normal atmospheric conditions.

In the SI system of units, exposure is measured in X unit: 1 X unit = 1 C/kg air, where C is Coulomb. The average energy dissipated to produce a single ion pair in air is 34 eV. Since the charge of an electron is equal to $1.6\text{x}10^{-19}$C, an association can be established between an X unit and the energy measured in Joules, dissipated in 1kg of air. Hence, one X unit is equivalent to 34 J/kg.

$$1\text{ X unit} = \frac{dQ}{dm} = 1\frac{\text{C}}{\text{kg air}}\times\frac{\text{ion pair}}{\text{e}}\times\frac{34\text{eV}}{\text{ion pair}} = 34\text{ J/kg}$$

1 R = 1 sC/cm^3**, Since** 1C = $3\text{x}10^9$sC, and the air density is 0.001293 g/cm^3 at STP, we find **1 Roentgen = absorbing energy of 0.00877 J/kg = 0.877 rad in dry air**.

1 X unit = 3880 R, 1 R = 2.58 x 10^{-4} C/kg

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Example: Medical X-ray shielding design is based on maximum weekly exposure of 100 mR for the control area and 10 mR for the un-controlled area, what are the corresponding exposure in SI unit? (Cember: example 6.1)

Answer: 25.8 μC/kg and 2.58 μC/kg

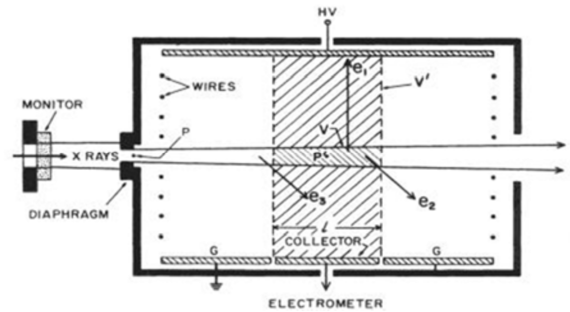
$$100\ \mathrm{mR} = 0.1\mathrm{R} \times 0.000258 \frac{\mathrm{C/kg}}{\mathrm{R}} = 25.8\ \mu C/\mathrm{kg}$$

$$10\ \mathrm{mR} = 0.01\mathrm{R} \times 0.000258 \frac{\mathrm{C/kg}}{\mathrm{R}} = 2.58\ \mu C/\mathrm{kg}$$

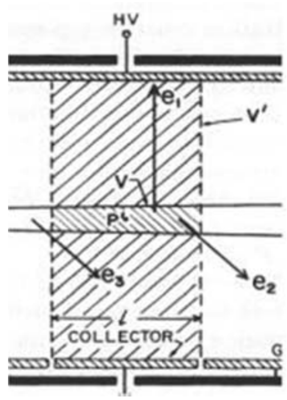
Free-air ion chamber

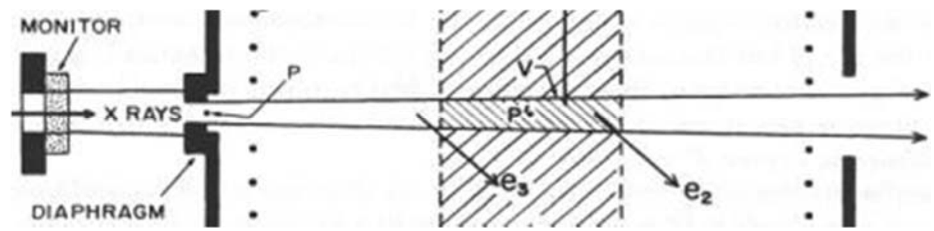
- The objective is to measure all the ionization produced by collision interactions in air by the electrons resulting from x-ray interactions in a known air mass, which is related to exposure. There are different designs of free-air chambers used in standardization laboratories in different countries, some cylindrical and some plane-parallel in geometry.
- The plane-parallel type, used at the NBS in calibrating cavity ion chambers for constant x-raytube potentials from 50 to 300 kV, is shown as follows.

Main components: Pb shielding box, diaphragm, plates parallel to the beam, Guard electrodes, and a set of wires provide a uniform electric field, The ionization for an exposure measurement is produced by electrons originating from volume V; the measured ionization is collected from V'.



- The lateral dimensions of the chamber are great enough to accommodate electrons like e1, which remain within V´ and thus produce all their ionization where it will be collected and measured.
- The electrons like e2, which originate within V, may have paths that carry some of their kinetic energy out of V´, but the remaining ionization they produce will go to the grounded guard plate instead of collector plate.
- This ionization is replaced by other electrons such as e3 that originate in the beam outside of volume V.
- Assumption: Volume V` as a whole is in **Charge Particle Equilibrium**.



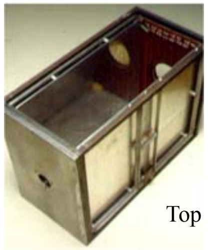


- The exposure at the aperture (point P) determined by the measurement must be corrected upward by the air attenuation between P and the midpoint P' in V • The volume of origin V can be replaced by a cylindrical volume $V_c = A_0 l$, where A_0 is the aperture of area, l is the path length of photon traversing V
- If Q (C) is the charge produced in V', the **exposure** at point P is

$$X = \frac{Q}{m}\exp(\mu x') = \frac{Q}{\rho A_0 l}\exp(\mu x')$$

where x' is the distance from P to P', and μ is the air attenuation coefficient.

The free air ionization chamber is the primary instrument for measuring exposure rates (R/hr) in an x-ray beam.

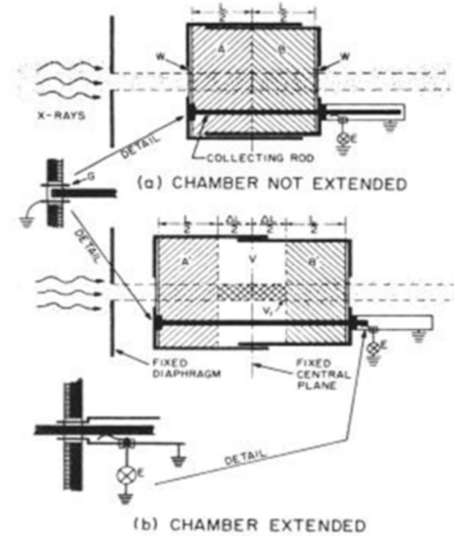


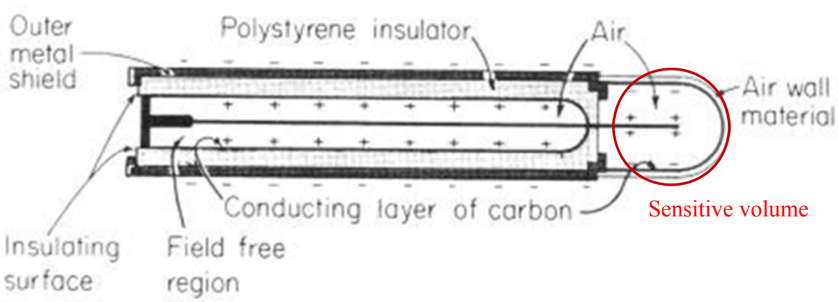
Top and right side covers removed

X-ray beam enters the chamber through the small opening on the front (right) of the unit. The electrical connector on lower left side of the unit is coupled to the anode.

Variable-length free-air chamber consists of two telescoping cylinders with the x-ray beam passing along their axis through holes at the centers of the two flat ends When the chamber is collapsed, electrons originating in the x-ray beam where it crosses the fixed central plane cannot reach the walls in any direction.

- Ionization measurement $Q1$ is made in the collapsed condition
- The chamber volume is expanded by a length ΔL (as much as 2-fold), while keeping the chamber midplane and the defining aperture fixed relative to the x-ray source; a second measurement $Q2$ is then made.
- The difference $Q1$-$Q2$ is due to electrons originating in V'





Example: Consider a chamber volume 2 cm^3, capacitance 5 pF, voltage before exposure to radiation 180 V, and voltage after the exposure 160 V after exposure time of 0.5 h. The radiation exposure and exposure rate are

1. $\Delta Q = C\Delta V =$ 100 pC
2. The exposure is 100 pC/(2 cm$^3\times 1.293\times 10^{-6}$ kg/cm^3) =38.67 μC/g = 0.150 R
3. The exposure rate is 150 mR/0.5h = 300 mR/h.

TABLE 12.1 First Atomic Ionization Potentials E_1 and W-Values in Several Gases for Electrons (W_e) and for 5-*MeV* α-Particles (W_α)

Gas	E_1 (eV)[b]	W_e (eV/i.p.)[a]	$\frac{E_1}{W_e}$	W_α (eV/i.p.)[a]	$\frac{W_\alpha}{W_e}$
He	24.6	41.3	0.60	42.7	1.034
Ne	21.6	35.4	0.61	36.8	1.040
Ar	15.8	26.4	0.60	26.4	1.000
Kr	14.0	24.4	0.57	24.1	0.988
Xe	12.1	22.1	0.55	21.9	0.991
H_2	15.4	36.5	0.42	36.43	0.998
N_2	15.6	34.8	0.45	36.39	1.046
O_2	12.1	30.8	0.39	32.24	1.047
CO_2	13.8	33.0	0.42	34.21	1.037

[a] Excerpted from ICRU (1979*b*). Reproduced with permission.
[b] Excerpted from the *Handbook of Chemistry and Physics*, 64th edition, CRC Press, Inc. (1983). Reproduced with permission.

Exposure-Dose relation:

Typical muscle tissue has a specific gravity of 1 and its elementary composition of hydrogen/oxygen:/nitrogen/carbon is 5.98:2.75:0.172:0.602×10^{22} atoms per gram respectively. Thus the **electron density is 3.28×10^{23} electrons/gm**. For air, its density is 1.293 mg/cm^3, and its electron density is **3.01×10^{23} electrons/gm**. The energy absorption of an exposure of 1 C/kg in tissue is $\frac{3.28}{3.01} \times 34 = 37$ J/kg

1 R = 1 sC/cm^3**, Since** 1C = 3x10^{9}sC, and the air density is 0.001293 g/cm^3 at STP, we find 1 Roentgen corresponds to absorbing energy of 0.00877 J/kg in dry air, or 0.0877×3.28/3.01**= 0.954 cGy** ≈ 1 rad in tissue!

kerma is the product of the photon energy fluence and the mass energy-transfer coefficient

$$K = E\,\Phi\,\frac{\mu_{tr}}{\rho}$$

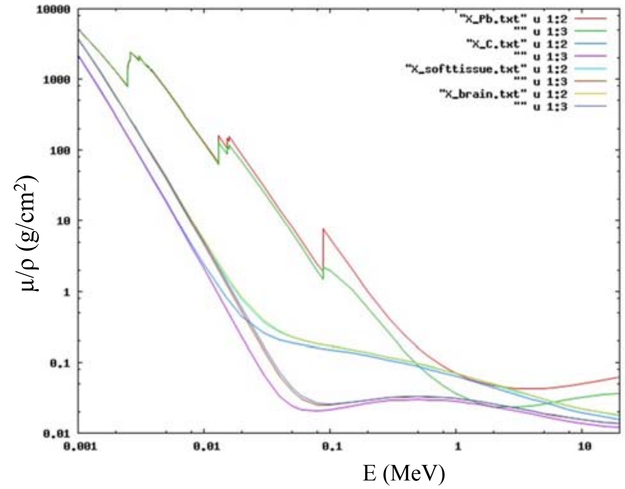
where, μ_{tr} is the mass energy-transfer coefficient is calculated using the total cross sections of those photon-matter interactions through which the photon transfers energy to the medium.

$$I = I_0 e^{-\mu t} = I_0 e^{-(\mu/\rho)x}$$

$$\Phi = \Phi_0 e^{-\mu t}$$

$$K = E\frac{d\Phi}{dx} = E\Phi\frac{\mu_{tr}}{\rho}$$

$$D = E\Phi\frac{\mu_{en}}{\rho}$$



Examples: A collimated beam of 0.3 MeV γ with energy flux 5 J/(m^{2}s) is shield by 2 cm Pb.
a) What is the incident particle flux in photons/(cm^{2}s)?
b) What is the exposure rate, in mR/h and C/kg/h, for the incident and exit beams?
c) What is the dose rate, mGy/h, in the incident and exit beam?

Exposure-Dose relation for Gamma ray:

The **exposure** of the radiation in **air** is $X = E\Phi\frac{\mu_a}{\rho_a} \times \frac{\text{C/kg}}{34\ \text{J/kg}}$

The **absorbed dosage (in Gy)** in soft tissue is $D = E\Phi\frac{\mu_m}{\rho_m} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times X$

The **absorbed dosage (in rad)** in soft tissue is $rad = \frac{87.7}{100} \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times R$

The radiation dose absorbed from any given exposure is determined by the ratio of the mass absorption coefficient of the medium to that of the air. In the case of soft tissue, the ratio of dose to exposure is approximately constant for photon energy from 0.1 to 10 MeV. In this energy range, the Compton scattering dominates the photon-tissue interaction and the cross-section depends essentially on the electronic density.

At lower energy, the photo-electric effect is more important. The cross-section depends on Z of the atomic absorber. Bone, containing calcium 10% by weight, absorbs much more energy than soft tissue.

Examples: A collimated beam of 0.3 MeV γ with energy flux 5 J/(m^{2}s) is shield by 2 cm Pb.
a) What is the incident particle flux in photons/(cm^{2}s)?
b) What is the exposure rate, in mR/h and C/kg/h, for the incident and exit beams?
c) What is the dose rate, mGy/h, in the incident and exit beam?

Pb: μ/ρ= 0.4031 (attenuation) 0.2455 (absorption) cm^2/g; ρ= 11.35 g/cm^3.
Air: μ/ρ= 0.1067 (attenuation) 0.02872 (absorption) cm^2/g; ρ= 0.001293 g/cm^3.

a) The energy Flux is EΦ=5×10^{-4} J/(cm^{2}s). Thus

$$\Phi = \frac{5\times10^{-4}}{0.3(1.6\times10^{-13})} = 1.04\times10^{10}\ \text{photons/(cm}^2\text{s)}$$

b) For your homework

$$I = I_0 e^{-\mu t} = I_0 e^{-(\mu/\rho)x}$$

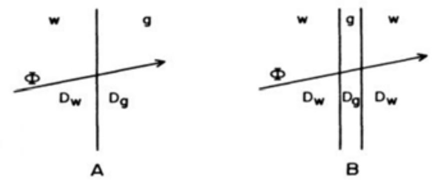
$$\Phi = \Phi_0 e^{-\mu t}$$

$$K = E\frac{d\Phi}{dx} = E\Phi\frac{\mu_{tr}}{\rho}$$

$$D = E\Phi\frac{\mu_{en}}{\rho}$$

Bragg-Gray principle

The amount of ionization produced in a small gas-filled cavity surrounded by a solid absorbing medium is proportional to the energy absorbed by the solid.



Let a Fluence Φ of radiation passes continuously across all interfaces, where

(A) A fluence Φ of charged particles crossing an interface between media *w* and *g*

(B) A fluence Φ of charged particles passing through a thin layer of medium *g* sandwiched between regions contain medium *w*.

The absorbed doses Dg, Dw on each side of the boundary are given by

$$D_g = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,g}\right]_T \sim D_w = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,w}\right]_T$$

Here (dT/ρdx) is the mass collision stopping power, evaluated at energy T. Thus we have

$$\frac{dT_m}{S_m dM_m} = \frac{dT_g}{S_g dM_g} \quad \text{or} \quad \frac{dE_m}{S_m dM_m} = \frac{dE_g}{S_g dM_g}$$

where S_m and S_g are mass stopping power of the medium and gas respectively.

Since $$\frac{dE_g}{dM_g} = w\left[\frac{\text{energy}}{\text{ion - pair}}\right] \times J[\#\text{ of ion - pair}]$$

Here, *w* is the energy per ion-pair, and *J* is the number of ion pairs.

The energy absorbed per unit mass in a medium is related to that of the gas by

$$\frac{dE_m}{dM_m} = \frac{S_m}{S_g} \times \frac{dE_g}{dM_g} = \frac{S_m}{S_g} \times w \times J$$

The **ratio** of mass stopping powers S_m to S_g of the medium and gas is listed below:

Energy (MeV)	graphite	water	tissue
0.411 (Au198)	1.032		
0.670 (Cs137)	1.027	1.162	1.145
1.25 (Co60)	1.017	1.155	1.137

Example: calculate the absorbed dose rate from the data on a tissue equivalent chamber with wall of equilibrium thickness embedded within a phantom and exposed to Co60 gamma rays for 10 minutes. The volume of the cavity is 1 cm^3, the capacitance is 5 pF, and the voltage decreases by 72 V due to γ-ray exposure.

1. Q=CΔV=5 pF × 72 V = 360 pC
2. The number of ion pairs is J=Q/e= 2.25 billions
3. Using Sm/Sg=1.137, w=34 eV/ion-pair, we find dEm/dMm=0.0108 Gy
4. The exposure is 10 minutes, we find the dose rate is 1.08 mGy/minute.

Specific gamma-ray strength

Defined as unit of C/(kg h) at 1 m from a 1 MBq source

$$\dot{X} = \Phi \frac{E\mu_{tr}}{\rho \times w} = \frac{f \times 1 \times 10^6 \times 3600\text{s/h}}{4\pi(1\text{m})^2} \times E \frac{\mu}{\rho \times 34 \frac{\text{J/kg}}{\text{C/kg}}}$$

X= exposure rate
f= fraction of transformation that result in a photon of energy E
E=energy of photon
μ= linear energy absorption coefficient m^{-1}.
ρ= density of air 1.293 kg/m^3.
w= mean energy dissipated in the production of an ion pair.

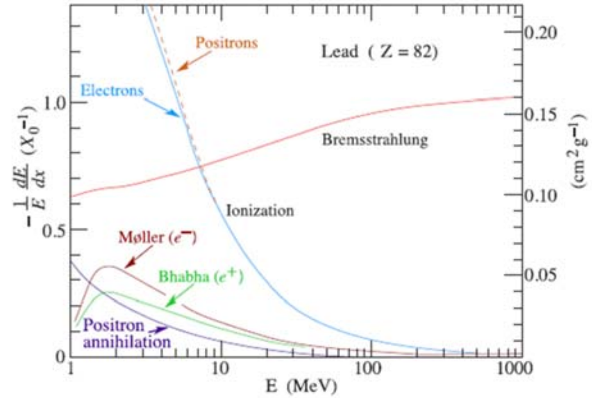
The source strength becomes $$\Gamma = 1.043 \times 10^{-6} \sum_i f_i \times E_i \times \mu_i \frac{(\text{C/kg})\,\text{m}^2}{(\text{MBq})\,\text{h}}$$

For photon source from 60 keV to 2 MeV, μ~0.0035 m^{-1}, nearly constant, the source is further simplified to:

$$\Gamma = 3.65 \times 10^{-9} \sum_i f_i \times E_i \frac{(\text{C/kg})\,\text{m}^2}{(\text{MBq})\,\text{h}} = 0.5 \sum_i f_i \times E_i \frac{(\text{R})\,\text{m}^2}{(\text{Ci})\,\text{h}}$$

Dosimetry calculation for electrons

PDG

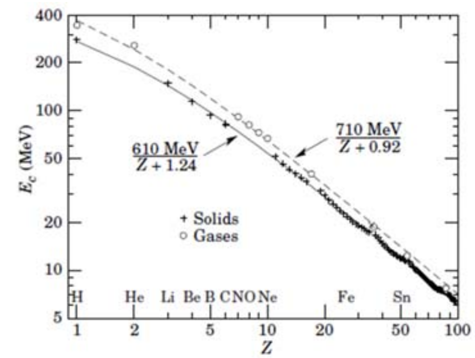


Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Møller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers, Pergamon Press, 1970. Messel and Crawford use X_0(Pb) = 5.82 g/cm^2, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials (X_0(Pb) = 6.37 g/cm^2).

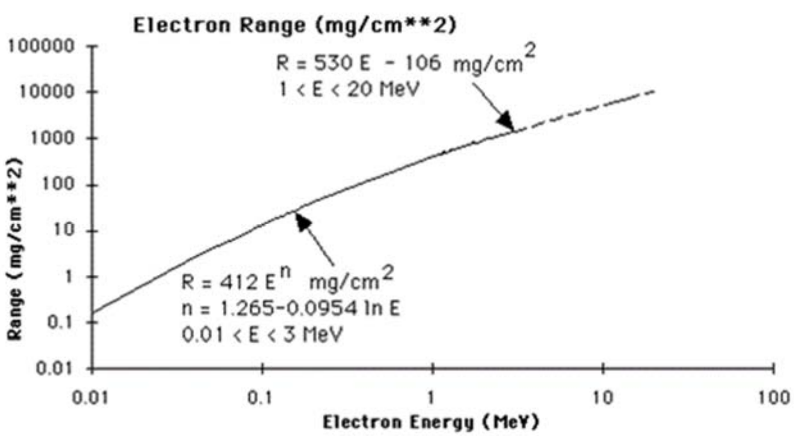
Critical energy for electrons and positrons

An electron (positron) loses energy by bremsstrahlung at a rate very nearly proportional to its energy, while collision losses (ionization and excitation) vary only slowly. The bremsstrahlung stopping power asymptotically approaches X_0E, where X_0 is the radiation length in the material and E is the particle's energy. The "Critical energy" is variously defined as (1) the energy at which the collision loss rate equals the bremsstrahlung rate (EGS4) or (2) the energy at which the collision loss rate equals X_0E (Rossi).

Z	Element	EGS4	Rossi
3	Lithium	149.06	149.06
14	Silicon	40.05	40.19
29	Copper	19.63	19.42
47	Silver	12.57	12.36
82	Lead	7.79	7.43



Electron critical energy for the chemical elements, using Rossi's definition. The fits shown are for solids and liquids (solid line) and gases (dashed line). The rms deviation is 2.2% for the solids and 4.0% for the gases.

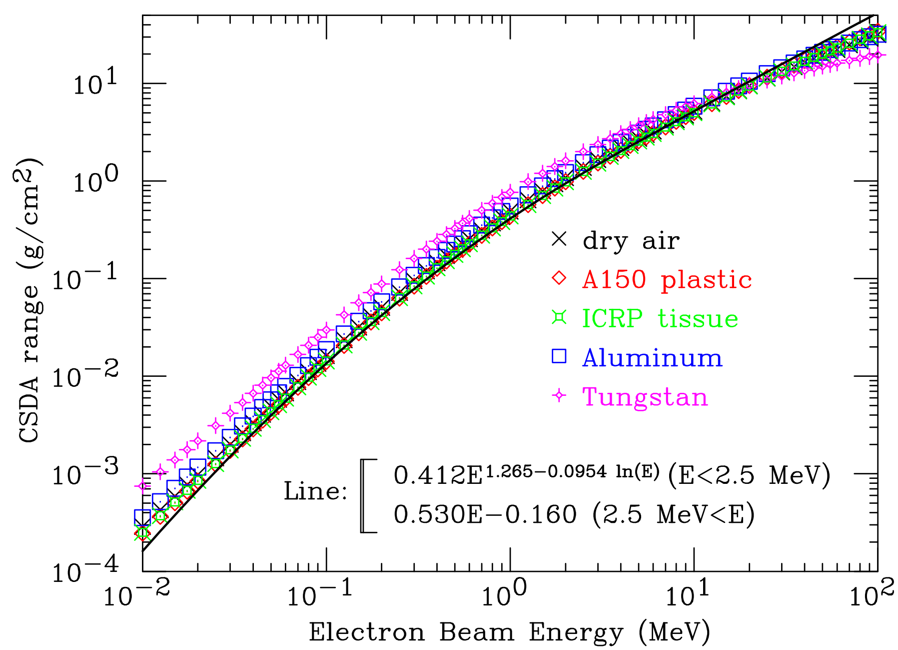


$R = 412E^{1.265-0.0954\ \ln E}$ For 0.01≤E≤2.5 MeV

$\ln E = 6.63 - 3.2376\,(10.2146 - \ln R)$ For R≤1200

$R = 530E - 106$ (E>2.5 MeV, R>1200)

R is the range in [mg/cm^2], and E is the maximum energy of beta-decay [MeV]. A useful rule of thumb for the range of beta particle is R[mg/cm^2] ~ 500 E [MeV].



A 2 MeV electron beam is used to irradiate a sample of plastic at thickness 0.5 g/cm^2. The 250 μA beam pass through a port of 10 cm in diameter. What is the absorbed dose?

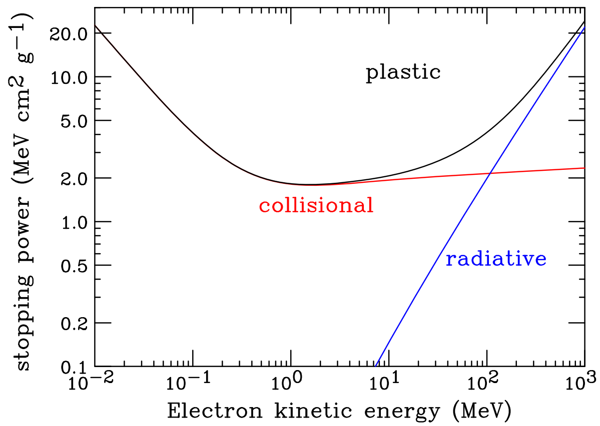
First check:

E (MeV)	stop-power	CSDA range	(in what unit?)
2	1.807	0.9971	

$$\Delta E = S\Delta x = S\rho dt \approx 0.90\ \mathrm{MeV}$$

$$D = \frac{S\rho dt}{M} = \frac{S\rho dt}{\rho dt A} = \frac{S}{A}$$

Why the dose is independent of the sample thickness?



Beta-radiation

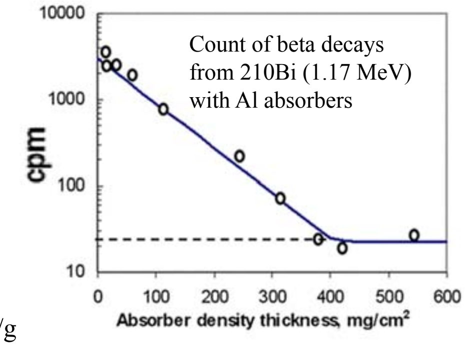
The intensity at depth can be approximated by

$$\varphi = \varphi_0 \exp(-\mu_\beta \ell)$$

For maximum energy E_m of the beta-ray in MeV, the absorption coefficients in air and tissue is

$$\mu_{\beta\,\mathrm{air}} = 16(E_m - 0.036)^{-1.4}\,\mathrm{cm^2/g}$$

$$\mu_{\beta\,\mathrm{tissue}} = 18.6(E_m - 0.036)^{-1.37}\,\mathrm{cm^2/g}$$



Dose from surface contamination

If the contamination area is a surface, one can calculate the energy flux as follows:

$$\varphi_0(E) = C_a[\mathrm{Bq/cm^2}]\overline{E}_\beta\, f_{\mathrm{direction}}\, f_{\mathrm{backscattering}}$$

Here Ca is the surface activity in [Bq/cm^2], the average energy of beta-decay is 1/3 of the maximum betatron decay energy, $f_{\mathrm{direction}}$~0.5 for a plane surface, and $f_{\mathrm{backscattering}}$~1.25 if 25% is back-scattering onto the direction to be evaluated..

The surface dose rate is $\dot{D}_{\beta,\mathrm{surf}} = \varphi_0(E)\,\mu_\beta$

Here μ_β is the mass attenuation coefficient for the beta-particles.

Why?

$$\varphi = \varphi_0 \exp(-\mu_\beta x)$$

$$D = E\left|\frac{d\varphi}{dx}\right| = E\varphi\mu_\beta$$

Example: A solution of 32P is spilled and contaminates a large surface to an areal concentration of 37 Bq/cm^2. What is the estimated beta-ray-contact dose to the skin and dose rate at a height 1 m above the contaminated surface?

For 32P→β^-+ … , Em=1.71 MeV or the average beta-energy is 0.7 MeV, we find

$$\mu_{\beta\,\mathrm{air}} = 16(E_m - 0.036)^{-1.4} = 7.78\,\mathrm{cm^2/g}$$

$$\mu_{\beta\,\mathrm{tissue}} = 18.6(E_m - 0.036)^{-1.37} = 9.18\,\mathrm{cm^2/g}$$

$$\varphi_S = C_a[\mathrm{Bq/cm^2}]\overline{E}_\beta\, f_{\mathrm{direction}}\, f_{\mathrm{backscattering}} = 37\times0.7\times1.6\times10^{-13}\times0.5\times3600\,[\mathrm{J/cm^2h}]$$

$$\dot{D}_\beta = 37\times0.7\times1.6\times10^{-13}\times0.5\times3600\,[\mathrm{J/cm^2h}]\times e^{-7.78\times0.129}\times\mu_{\beta,\mathrm{tissue}}$$

$$= 2.5\times10^{-5}\,[\mathrm{Gy/h}] = 0.025\,[\mathrm{mGy/h}]$$

Example: A worker accidentally spill 3700 Bq of 32P solution over an area of 10 cm^2 on her skin. What is the dose rate at the contaminated skin?

For 32P→β⁻+ … , Em=1.71 MeV or the average beta-energy is 0.7 MeV, we find

$$\mu_{\beta\,\mathrm{air}} = 16(E_m - 0.036)^{-1.4} = 7.78\ \mathrm{cm^2/g}$$

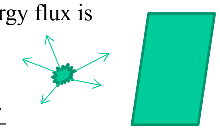
$$\mu_{\beta\,\mathrm{tissue}} = 18.6(E_m - 0.036)^{-1.37} = 9.18\ \mathrm{cm^2/g}$$

$$\varphi_S = (3700/10)\times 0.7\times 1.6\times 10^{-13}\times 0.5\times 3600\ [\mathrm{J/cm^2h}]$$

$$\dot{D}_\beta = 370\times 0.7\times 1.6\times 10^{-13}\times 0.5\times 3600\ [\mathrm{J/cm^2h}]\times \mu_{\beta,\mathrm{tissue}}$$
$$= 6.8\times 10^{-4}\ [\mathrm{Gy/h}] = 0.68\ [\mathrm{mGy/h}]$$

Submersion Dose

Consider a source with activity Ca in the entire space, the volume energy flux is



$$\varphi_V = C_a[\mathrm{Bq/cm^3}]\,\overline{E}_\beta$$

What is the surface flux?

$$\varphi_S = \tfrac{1}{2} C_a[\mathrm{Bq/cm^3}]\,\overline{E}_\beta\cdot\Delta\ell = \tfrac{1}{2}\frac{C_a[\mathrm{Bq/cm^3}]\,\overline{E}_\beta}{\rho_{\mathrm{air}}\mu_{\mathrm{air}}}$$

Here the factor of ½ is due to the fact that only half of the decay β will reach the surface. The dose to a person standing in the room is $\dot{D}_\beta = \varphi_S\ \mu_{\beta,\mathrm{tissue}}$

We note that the dose rate is independent of the surface area of a person typically S~1.7 m^2.

Example: Calculate the dose rate of a person immersed in a large cloud of 85Kr at the concentration of 37 kBq/m^3.

For 85Kr→β⁻+ … , Em=0.687 MeV or the average beta-energy is 0.229 MeV, we find

$$\mu_{\beta\,\mathrm{air}} = 16(E_m - 0.036)^{-1.4} = 29.2\ \mathrm{cm^2/g}$$

$$\mu_{\beta\,\mathrm{tissue}} = 18.6(E_m - 0.036)^{-1.37} = 33.5\ \mathrm{cm^2/g}$$

$$\varphi_V = (37000/10^6)\times 0.229\times 1.6\times 10^{-13}\times 3600\ [\mathrm{J/cm^3h}]$$

$$\varphi_S = 6.46\times 10^{-11}\ [\mathrm{J/cm^2h}]$$

$$\dot{D} = \varphi_S\,\mu_{\beta,\mathrm{tissue}} = 2.16\times 10^{-6}\ \ [\mathrm{Gy/h}]$$

Neutrons

$$I = I_0 e^{-\sigma N t} = I_0 e^{-\mu\cdot\rho t} = I_0 e^{-\mu\cdot x}$$
$$\mu = \sigma N$$

Here

1) σ is the cross-section
2) N is the number of particles per unit mass
3) μ is the mass attenuation of the mediu
4) x is the target thickness in mass/area.

The dose rate of the neutrons becomes $\dot{D} = E\phi(E)\sum \mu_i = E\phi(E)\sum N_i\sigma_i f_i$

Here ϕ is the neutron flux in neutrons/(cm^2s), E is energy of neutrons, N_i is the number of atoms per kg for the i-th element, σ_i is the cross-section, and f is the mean fractional energy transfer per collision. For fast neutrons E>100 keV, important interaction between neutron and atoms is elastic collision. The energy transfer is

$$f = \left\langle 1 - \frac{E}{E_0}\right\rangle = \left\langle \left(1 - \left(\frac{M-m}{M+m}\right)^2\right)\cos^2\theta \right\rangle = \frac{2Mm}{(M+m)^2}$$

Example: What is the absorbed dose rate to soft tissue in a beam of 5 MeV neutrons with intensity 2000 per square centimeters per second?

Element	By mass	N/kg (10^{23})	f	σ (b)	Nσf
Oxygen	71.39%	269	.111	1.55	4.628
Carbon	14.89%	64.1	.142	1.65	1.502
Hydrogen	10%	598	.5	1.50	44.85
Nitrogen	3.47%	14.9	.124	1.00	0.185
Sodium	0.15%	0.393	.08	2.3	0.007
Chlorine	0.1%	0.17	.053	2.8	0.003

5 MeV neutron specific $\sum N_i\sigma_i f_i = 51.17\ \ \mathrm{cm^2/kg}$

$$\dot{D} = E\phi(E)\sum N_i\sigma_i f_i = 2000\times(5\times 1.6\times 10^{-13})\times 51.17 = 8.19\times 10^{-8}\ \mathrm{Gy/s} = 295\,\mu\mathrm{Gy/h}$$

For thermal neutrons, there are sizable nuclear reactions such as (n,p) and (n,γ) reactions,. The dose rate becomes

$$\dot{D} = \phi(E)\sum Q_i N_i\sigma_i$$

Here Q_i is the Q-value (or energy release) of the neutron reaction. The cross-section and Q values are σ(14N(n,p))=1.84 b, Q=0.63 MeV; and σ(1H(n,γ))=0.33 b, Q=2.2 MeV